Part 1 Test FRQ

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. The Better Business Council of a large city has concluded that students in the city's schools are not learning enough about economics to function in the modern world. These findings were based on test results from a random sample of 20 twelfth-grade students who completed a 46-question multiple-choice test on basic economic concepts. The data set below shows the number of questions that each of the 20 students in the sample answered correctly.

12	16	18	17	18	33	41	44	38	35
19	36	19	13	43	8	16	14	10	9

(a)   Display these data in a stemplot.

(b)   Use your stemplot from part (a) to describe the main features of this score distribution. (Be sure to include the context of the problem in your description.)

(c)   Why would it be misleading to report only a measure of center for this score distribution? (Be sure to include the context of the problem in your answer.)

2. A large regional real estate company keeps records of home sales for each of its sales agents. Each month the company publishes the sales volume for each agent. Monthly sales volume is defined as the total sales price of all homes sold by the agent during a month. The figure below displays the cumulative relative frequency plot of the most recent monthly sales volume (in hundreds of thousands of dollars) for these agents.

(a)   In the <u>context</u> of this question, explain what information is conveyed by the circled point.

(b)   What proportion of sales agents achieved monthly sales volumes between $700,000 and $800,000? (Show your work)

(c)   For values between 10 and 11 on the horizontal axis, the cumulative relative frequency plot is
flat.   In the <u>context</u> of this question, explain what this means.

(d)   A bonus is to be given to 20 percent of the sales agents. Those who achieved the highest monthly sales volume during the preceding month will receive a bonus. What is the minimum monthly sales volume an agent must have achieved to qualify for the bonus? Justify your answer.

3. To determine the amount of sugar in a typical serving of breakfast cereal, a student randomly selected 60 boxes of different types of cereal from the shelves of a large grocery store.

The student noticed that the side panels of some of the cereal boxes showed sugar content based on one-cup servings, while others showed sugar content based on three-quarter-cup servings. Many of the cereal boxes with side panels that showed three-quarter-cup servings were ones that appealed to young children, and the students wondered whether there might be some difference in the sugar content of the cereals that showed different-size servings on their side panels. To instigate the question, the data were separated into two groups. One group consisted of 29 cereals that showed one-cup serving sizes; the other group consisted of 31 cereals that showed three-quarter-cup serving sizes. The boxplots shown below display sugar content (in grams) per serving of the cereals for each of the two serving sizes.
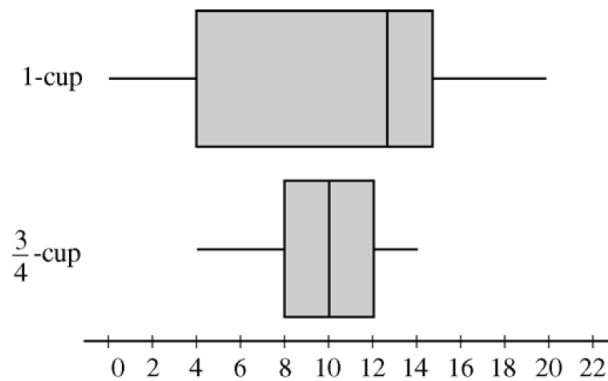
(a) Write a few sentences to compare the distributions of sugar content per serving for the two serving sizes of cereals. (Include the context in your descriptions.)

After analyzing the boxplots on the previous page, the students decided that instead of a comparison of sugar content per recommended serving, it might be more appropriate to compare sugar content for equal-size servings. To compare the amount of sugar in serving sizes of one cup each, the amount of sugar in each of the cereals showing three-quarter-cup servings on their side panels was multiplied by $\frac{4}{3}$. The bottom boxplot shown below displays sugar content (in grams) per cup for those cereals that showed a serving size of three-quarter-cup on their side panels.

Adjusted Sugar Content Per Cup
(grams)
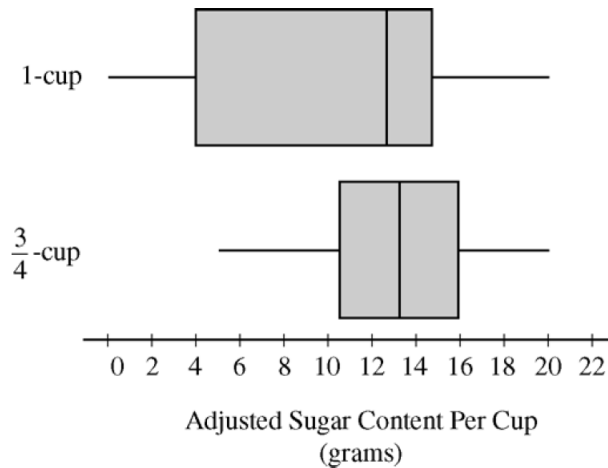
(b)　　What <u>new information</u> about sugar content do the boxplots above provide? (Compared with the previously displayed distributions.)

(c)　　Based on the boxplots shown above on this page, how would you expect the mean amounts of sugar per cup to compare for the different recommended serving sizes? Justify your answer.

Part 1 Test Retake FRQ

1.

As gasoline prices have increased in recent years, many drivers have expressed concern about the taxes they pa[y]
on gasoline for their cars. In the United States, gasoline taxes are imposed by both the federal government and
by individual states. The boxplot below shows the distribution of the state gasoline taxes, in cents per gallon,
for all 50 states on January 1, 2006.

State Gasoline Taxes (in cents per gallon)
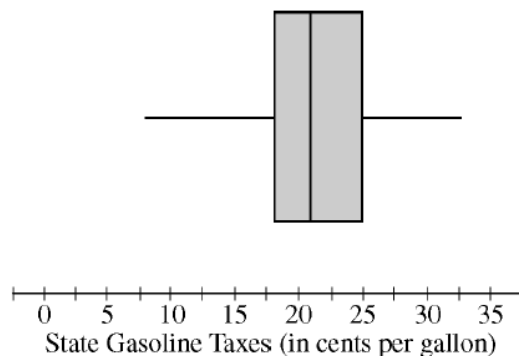
(a) Based on the boxplot, what are the approximate values of the median and the interquartile range of the
distribution of state gasoline taxes, in cents per gallon? Mark and label the boxplot to indicate how you
found the approximated values.

(b) The federal tax imposed on gasoline was 18.4 cents per gallon at the time the state taxes were in effect.
The federal gasoline tax was added to the state gasoline tax for each state to create a new distribution of
combined gasoline taxes. What are approximate values, in cents per gallon, of the median and interquartile
range of the new distribution of combined gasoline taxes? Justify your answer.

2. A large regional real estate company keeps records of home sales for each of its sales agents. Each month the company publishes the sales volume for each agent. Monthly sales volume is defined as the total sales price of all homes sold by the agent during a month. The figure below displays the cumulative relative frequency plot of the most recent monthly sales volume (in hundreds of thousands of dollars) for these agents.

(a)   In the context of this question, explain what information is conveyed by the circled point.

(b)   What proportion of sales agents achieved monthly sales volumes over 1 million dollars? (Answer in context and show your work.)

(c)   Between the values of 2 and 3 on the horizontal axis is the steepest section of the relative frequency plot. In the context of this question, explain what this means.

(d)   In the context of this problem, between which two values was the Monthly Sales Volume for the sales associates in the 2nd Quartile (from 25% to 50%).

3.

As a part of the United States Department of Agriculture's Super Dump cleanup efforts in the early 1990s, various sites in the country were targeted for cleanup. Three of the targeted sites—River X, River Y, and River Z—had become contaminated with pesticides because they were located near abandoned pesticide dump sites. Measurements of the concentration of aldrin (a commonly used pesticide) were taken at twenty randomly selected locations in each river near the dump sites.

The boxplots shown below display the five-number summaries for the concentrations, in parts per million (ppm) of aldrin, for the twenty locations that were sampled in each of the three rivers.

River X

River Y

River Z

3   4   5   6   7   8   9
Concentrations of Aldrin (ppm)
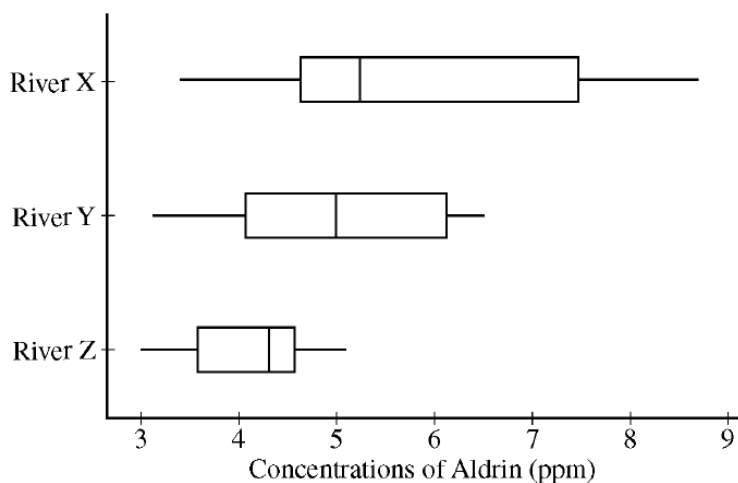
(a) Compare the distributions of the concentration of aldrin among the three rivers.

(b) The twenty concentrations of aldrin for River X are given below.

3.4   4.0   5.6   3.7   8.0   5.5   5.3   4.2   4.3   7.3
8.6   5.1   8.7   4.6   7.5   5.3   8.2   4.7   4.8   4.6

Construct a stemplot that displays the concentrations of aldrin for River X.

(c) Describe a characteristic of the distribution of aldrin concentrations in River X that can be seen in the stemplot but cannot be seen in the boxplot.

Part 3 Test FRQ

1. Alzheimer's disease results in a loss of cognitive ability beyond what is expected with typical aging. A local newspaper published an article with the following headline.

> Study Finds Strong Association Between Smoking and Alzheimer's

The article reported that a study tracked the medical histories of 21,123 men and women for 23 years. The article stated that, for those who smoked at least two packs of cigarettes a day, the risk of developing Alzheimer's disease was 2.57 times the risk for those who did not smoke.

(a) Identify the explanatory and response variables in the study

Explanatory variable:

Response variable:

(b) Is the study described in the article an observational study or an experiment? Explain.

(c) Exercise status (regular weekly exercise versus no regular weekly exercise) was mentioned in the article as a possible confounding variable. Explain how exercise status could be a confounding variable in the study.

2. At a certain university, students who live in the dormitories eat at the common dining hall. Recently, some students have been complaining about the quality of the food served there. The dining hall manager decided to do a survey to estimate the proportion of students living in the dormitories who think that the quality of the food should be improved. One evening, the manager asked the first 100 students entering the dining hall to answer the following question.

Many students believe that the food served in the dining hall needs improvement. Do you think that the quality of food served here needs improvement, even though that would increase the cost of the meal plan?

    ____ Yes   ____ No   ____ No opinion

a) In this setting, explain how bias may have been introduced based on the way this convenience sample was selected <u>and</u> suggest how the sample could have been selected differently to avoid that bias.

b) In this setting, explain how bias may have been introduced based on the way the question was worded <u>and</u> suggest how it could have been worded differently to avoid that bias.

3. A manufacturer of toxic pesticide granules plans to use a dye to color the pesticide so that birds will avoid eating it. A series of experiments will be designed to find colors or patterns that three bird species (blackbirds, starlings, and geese) will avoid eating. Representative samples of birds will be captured to use in the experiments, and the response variable will be the amount of time a hungry bird will avoid eating food of a particular color or pattern.

    (a) Previous research has shown that male birds do not avoid solid colors. However, it is possible that males might avoid colors displayed in a pattern, such as stripes. In an effort to prevent males from eating the pesticide, the following two treatments are applied to the pesticide granules.

        Treatment 1: A red background with narrow blue strips.

        Treatment 2: A blue background with narrow red strips.

        To increase the experiment's ability to detect a difference in the two treatments in the analysis of the experiment, the researcher decided to block on the three species of birds (blackbirds, starlings, and geese). Assuming there are 20 birds of each of the three species, explain how you would assign birds to treatments in such a block design.

Other than blocking, what could the researcher do to increase the experiment's ability to detect a difference in the two treatments in the analysis of the experiment? Explain how your approach would increase the experiment's ability to detect a difference.

Part 4 Test FRQ

1. An automobile company wants to learn about customer satisfaction among the owners of five specific car models. Large sales volumes have been recorded for three of the models, but the other two models were recently introduced so their sales volumes are smaller. The number of new cars sold in the last six months for each of the models is shown in the table below.

Car Model	A	B	C	D	E	Total
Number of new cars sold in the last six months	112,338	96,174	83,241	3,278	2,323	297,354

The company can obtain a list of all individuals who purchased new cars in the last six months for each of the five models shown in the table. The company wants to sample 2,000 of these owners.

(a) For the simple random sample of 2,000 new car owners, what is the expected number of owners of model E and the standard deviation of the number of owners of model E?

(b) When selecting a simple random sample of 2,000 new car owners, how likely is it that fewer than 12 owners of model E would be included in the sample? Justify your answer.

(c) The company is concerned that a simple random sample of 2,000 owners would include fewer than 12 owners of model D and fewer than 12 owners of model E. Briefly describe a sampling method for randomly selecting 2,000 owners that will ensure at least 12 owners will be selected for each of the 5 car models.

2. A professional sports team evaluates potential players for a certain position based on two main characteristics, speed and strength.

(a) Speed is measured by the time required to run a distance of 40 yards, with smaller times indicating more desirable (faster) speeds. From previous speed data for all players in this position, the times to run 40 yards have a mean of 4.60 seconds and a standard deviation of 0.15 seconds, with a minimum time of 4.40 seconds, as shown in the table below.

	Mean	Standard Deviation	Minimum
Time to run 40 yards	4.60 seconds	0.15 seconds	4.40 seconds

Based on the relationship between the mean, standard deviation, and minimum time, is it reasonable to believe that the distribution of 40-yard running times is approximately normal? Explain.

(b) Strength is measured by the amount of weight lifted, with more weight indicating more desirable (greater) strength. From previous strength data for all players in this position, the amount of weight lifted has a mean of 310 pounds and a standard deviation of 25 pounds, as shown in the table below.

	Mean	Standard Deviation
Amount of weight lifted	310 pounds	25 pounds

Calculate and interpret the z-score for a player in this position who can lift a weight of 370 pounds.

(c) The characteristics of speed and strength are considered to be of equal importance to the team in selecting a player for the position. Based on the information about the means and standard deviations of the speed and strength data for all players and the measurements listed in the table below for Players A and B, which player should the team select if the team can only select one of the two players? Justify your answer.

	Player A	Player B
Time to run 40 yards	4.42 seconds	4.57 seconds
Amount of weight lifted	370 pounds	375 pounds

3. The ELISA tests whether a patient has contracted HIV. The ELISA is said to be positive if it indicates that HIV is present in the blood sample, and the ELISA is said to be negative if it does not indicate that HIV is present in the blood sample. Instead of directly measuring the presence of HIV, the ELISA measures levels of antibodies in the blood that should be elevated if HIV is present. Because of variability in antibody levels among human patients, the ELISA does not always indicate the correct result.

As part of a training program, staff at a testing lab applied the ELISA to 500 blood samples known to contain HIV. The ELISA was positive for 489 of those blood samples and negative for the other 11 samples. As part of the same training program, the staff also applied the ELISA to 500 other blood samples known to not contain HIV. The ELISA was positive for 37 of those blood samples and negative for the other 463 samples.

(a) When a new blood sample arrives at the lab, it will be tested to determine whether HIV is present. Using the data from the training program, estimate the probability that the ELISA would be positive when it is applied to a blood sample that does not contain HIV.

(b) Among the blood samples examined in the training program that provided positive ELISA results for HIV, what proportion actually contained HIV?

(c) When a blood sample yields a positive ELISA result, two more ELISAs are performed on the same blood sample. If at least one of the two additional ELISAs is positive, the blood sample is subjected to a more expensive and more accurate test to make a definitive determination of whether HIV is present in the sample. Repeated ELISAs on the same sample are generally assumed to be independent. Under the assumption of independence, what is the probability that a new blood sample that comes into the lab will be subjected to the more expensive test if that sample does not contain HIV?

Part 4 Test Retake FRQ

1. A simple random sample of adults living in a suburb of a large city was selected. The age and annual income of each adult in the sample was recovered. The resulting data are summarized in the table below.

	Annual Income			
Age Category	$25,000-$35,000	$35,001-$50,000	Over $50,000	Total
21-30	8	15	27	50
31-45	22	32	35	89
46-60	12	14	27	53
Over 60	5	3	7	15
Total	47	64	96	207

(a) What is the probability that a person chosen at random from those in this sample will be in the 31-45 age category?

(b) What is the probability that a person chosen at random from those in this sample whose incomes are over $50,000 will be in the 31-45 age category? Show your work.

(c) Based on your answers to parts (a) and (b), is annual income independent of age category for those in this sample? Explain.

2. An airline claims that there is a 0.08 probability that a coach-class ticket holder who flies frequently will be upgraded to first class on any flight. This outcome is independent from flight to flight. Chelsea is a frequent flier who always purchases coach-class tickets.

(a) What is the probability that Chelsea will be upgraded between 3 and 5 times in the next 30 flights?

(b) What is the probability that Chelsea's first upgrade will occur on or before her fifth flight?

(c) Chelsea will take 156 flights next year. Would you be surprised if Chelsea receives more than 25 upgrades to first class during the year? Justify your answer.

3. Two antibiotics are available as treatment for a common ear infection in children.
- Antibiotic A is known to effectively cure the infection 60 percent of the time. Treatment with antibiotic A costs $50.
- Antibiotic B is known to effectively cure the infection 90 percent of the time. Treatment with antibiotic B costs $80.

The antibiotics work independently of one another. Both antibiotics can be safely administered to children. A health insurance company intends to recommend one of the following two plans of treatment for children with this ear infection.
- Plan I: Treat with antibiotic A first. If it is not effective, then treat with antibiotic B.
- Plan II: Treat with antibiotic B first. If it is not effective, then treat with antibiotic A.

(a) If a doctor treats with an ear infection using plan I, what is the probability that the child will be cured?

If a doctor treats with an ear infection using plan II, what is the probability that the child will be cured?

(b) Compute the expected cost per child when plan I is used for treatment.

Compute the expected cost per child when plan II is used for treatment.

(c) Based on the results in parts (a) and (b), which plan would you recommend? Explain your recommendation.