Name: Answers

STATISTICS

PART 6 PRACTICE EXAM

Time – 1 hour and 30 minutes

Number of multiple choice questions – 20

Number of free response questions - 3

1.

A study is conducted to estimate the mean family income in the eastern half of North Dakota by the construction of a 90% confidence interval. A pilot survey of 10 families and their income gives a sample standard deviation of \$800. Assuming that the population standard deviation is also about \$800, which of the following is the smallest sample size that will assure that the confidence interval's margin of error is at most \$100?

(A) 15

(B) 55

(C) 175

(D) 250

(E) 425

$ME = z^* \frac{\sigma}{\sqrt{n}}$

$100 = 1.645 \cdot \frac{800}{\sqrt{n}}$

$n = 173.19$ round up

2.

A team of biologists has collected data for an experiment on caloric intake of 28 lab rats. They used a one-sample t-test with $\alpha = 0.05$ and chose to run a two-sided test. Which of the following is the smallest possible test statistic that would reject the null hypothesis in favor of the alternative hypothesis?

(A) 1.253

(B) 1.701

(C) 1.703

(D) 2.012

(E) 2.301

$df = 28 - 1 = 27$

$p = tcdf(2.301, 9E99, 27) * 2$

$= .029$

less than α

3.
An automobile manufacturer claims that the average gas mileage of a new model is 35 miles per gallon (mpg). A consumer group is skeptical of this claim and thinks the manufacturer may be overstating the average gas mileage. If μ represents the true average gas mileage for this new model, which of the following gives the null and alternative hypotheses that the consumer group should test?

they think it is less

(A) ~~H_0: $\mu \le 35$ mpg~~
~~H_a: $\mu \geq 35$ mpg~~

(B) ~~H_0: $\mu \leq 35$ mpg~~
~~H_a: $\mu > 35$ mpg~~

Ho is always $\mu =$ something

(C) H_0: $\mu = 35$ mpg
H_a: $\mu > 35$ mpg

(D) H_0: $\mu = 35$ mpg
H_a: $\mu < 35$ mpg

(E) H_0: $\mu = 35$ mpg
H_a: $\mu \neq 35$ mpg

4.
In a test of the hypothesis $H_0: \mu = 100$ *versus* $H_a: \mu > 100$, the power of the test when $\mu = 101$ would be greatest for which of the following choices of sample size n and significance level α ?

high n high α

(A) $n = 10$, $\alpha = 0.05$

(B) $n = 10$, $\alpha = 0.01$

(C) $n = 20$, $\alpha = 0.05$

(D) $n = 20$, $\alpha = 0.01$

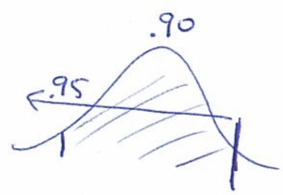
(E) It cannot be determined from the information given.

5.
A random sample of the costs of repair jobs at a large muffler repair shop produces a mean of \$127.95 and a standard deviation of \$24.03. If the size of this sample is 40, which of the following is an approximate 90 percent confidence interval for the average cost of a repair at this repair shop?

(A) ~~\$127.95 ± \$4.87~~
(B) \$127.95 ± \$6.25
(C) ~~\$127.95 ± \$7.45~~
(D) \$127.95 ± \$30.81
(E) \$127.95 ± \$39.53

$$127.95 \pm t^*_{39} \frac{24.03}{\sqrt{40}}$$

invT(.95, 39)



$$127.95 \pm 1.685 \frac{2403}{\sqrt{40}}$$

$$127.95 \pm 6.40$$

6.

The Attila Barbell Company makes bars for weight lifting. The weights of the bars are independent and are normally distributed with a mean of 720 ounces (45 pounds) and a standard deviation of 4 ounces. The bars are shipped 10 in a box to the retailers. The weights of the empty boxes are normally distributed with a mean of 320 ounces and a standard deviation of 8 ounces. The weights of the boxes filled with 10 bars are expected to be normally distributed with a mean of 7,520 ounces and a standard deviation of

The Pythagorean Theorem of Statistics.

$$\sqrt{\underbrace{4^2+4^2+4^2+\dots+4^2}_{10}+8^2}$$

$$=\sqrt{4^2\cdot 10+8^2} = \boxed{\sqrt{224}}$$

(A) $\sqrt{12}$ ounces
(B) $\sqrt{80}$ ounces
(C) $\sqrt{224}$ ounces
(D) 48 ounces
(E) $\sqrt{1,664}$ ounces

7.

Which of the following is a criterion for choosing a t-test rather than a z-test when making an inference about the mean of a population?

(A) The standard deviation of the population is unknown.
(B) The mean of the population is unknown.
(C) The sample may not have been a simple random sample.
(D) The population is not normally distributed.
(E) The sample size is less than 100.

8.

A student working on a history project decided to find a 95 percent confidence interval for the difference in mean age at the time of election to office for former American Presidents versus former British Prime Ministers. The student found the ages at the time of election to office for the members of both groups, which included all of the American Presidents and all of the British Prime Ministers, and used a calculator to find the 95 percent confidence interval based on the t-distribution. This procedure is not appropriate in this context because

(A) the sample sizes for the two groups are not equal

(B) the entire population was measured in both cases, so the actual difference in means can be computed and a confidence interval should not be used

(C) elections to office take place at different intervals in the two countries, so the distribution of ages cannot be the same

(D) ages at the time of election to office are likely to be skewed rather than bell-shaped, so the assumptions for using this confidence interval formula are not valid

(E) ages at the time of election to office are likely to have a few large outliers, so the assumptions for using this confidence interval formula are not valid

9.

Two manufacturers of canned goods add different amounts of water to their canned vegetables. For a 15.25-oz can of vegetables, one manufacturer adds a mean of 4.5 oz with a standard deviation of 0.63 oz. The other manufacturer adds a mean of 5.1 oz with a standard deviation of 0.57 oz. What are the mean and standard deviation for the difference in the amount of water added? (Assume independence for the manufacturers.)

(A) mean 0.6 oz; standard deviation 0.06 oz

(B) mean 0.6 oz; standard deviation 0.85 oz

(C) mean 0.6 oz; standard deviation 0.072 oz

~~(D)~~ mean 9.6 oz; standard deviation 0.06 oz

~~(E)~~ mean 9.6 oz; standard deviation 1.20 oz

mean: $5.1 - 4.5 = .6$

Pythagorean Theorem of Statistics.

$sd = \sqrt{.63^2 + .57^2}$

$= .85$

10.

Which of the following is *not* a characteristic for t-distributions?

(A) The t-distributions are mound-shaped.

(B) The t-distributions are centered at 0.

(C) The t-distributions have more area in the tails than a normal distribution.

(D) The t-distributions use s as an estimate of σ.

(E) As the number of degrees of freedom decreases, the t-distribution approaches the normal distribution. increases.

11.

A t-distribution with 30 degrees of freedom is an appropriate statistical model when

(A) constructing a confidence interval based on a random sample of size 29.

(B) constructing a confidence interval based on two independent random samples of sizes 13 and 17.

(C) using a t-statistic based on a random sample of size 30.

(D) using a t-statistic based on a random sample of size 31. $31 - 1 = 30$

(E) we do not know μ, σ, or the sample size.

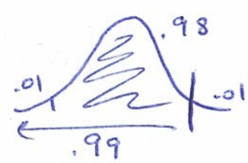
12.

The weekly beef consumption for a random sample of 15 adults is 1.25 lb with a standard deviation of 0.39 lb. A modified boxplot of the data reveals a slight skew with no outliers. Find a 98% confidence interval for the average weekly beef consumption of adults.

$1.25 \pm t_{14} \dfrac{0.39}{\sqrt{15}}$

(A) ~~$0.39 \pm 2.326\dfrac{1.25}{\sqrt{15}}$~~

(B) ~~$0.39 \pm 2.624\dfrac{1.25}{\sqrt{15}}$~~



invT(.99, 14)

2.624

(C) $1.25 \pm 2.326\dfrac{0.39}{\sqrt{15}}$

(D) $1.25 \pm 2.602\dfrac{0.39}{\sqrt{15}}$

(E) $1.25 \pm 2.624\dfrac{0.39}{\sqrt{15}}$

13.

one sided.

A botanist is interested in testing H_0: $\mu = 3.5$ cm versus H_a: $\mu > 3.5$, where μ = the mean petal length of one variety of flowers. A random sample of 50 petals gives significant results at a 5 percent level of significance. Which of the following statements about the confidence interval to estimate the mean petal length is true?

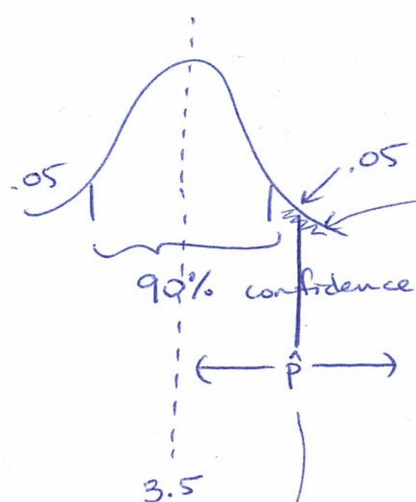
(A) The specified mean length of 3.5 cm is within a 90 percent confidence interval.

(B) The specified mean length of 3.5 cm is not within a 90 percent confidence interval.

(C) The specified mean length of 3.5 cm is below the lower limit of a 90 percent confidence interval.

(D) The specified mean length of 3.5 cm is below the lower limit of a 95 percent confidence interval.

(E) Not enough information is available to answer the question.



$\hat{p}$ not in the interval, it is in the upper tail

14.

A random sample of 16 light bulbs of one brand was selected to estimate the mean lifetime of that brand of bulbs. The sample mean was 1,025 hours, with a standard deviation of 130 hours. Assuming that the lifetimes are approximately normally distributed, which of the following will give a 95 percent confidence interval to estimate the mean lifetime?

$t^*_{15} = \text{invT}(.975, 15)$

$= 2.13$

(A) ~~$1{,}025 \pm 1.96\sqrt{\frac{130}{16}}$~~

(B) ~~$1{,}025 \pm 1.96\sqrt{130}$~~

(C) ~~$1{,}025 \pm 1.96\frac{130}{\sqrt{16}}$~~

(D) $1{,}025 \pm 2.13\frac{130}{\sqrt{16}}$ s. $\sqrt{n}$

(E) $1{,}025 \pm 2.13\sqrt{\frac{130}{16}}$

15.

Two random samples from two independent populations are taken with the following results.

Sample 1	Sample 2
$n = 30$	$n = 40$
$\bar{x} = 26$	$\bar{x} = 31$
$s = 3.2$	$s = 3.8$

The standard error of the sampling distribution of the differences of the means is

(A) 0.594

(B) 0.838

(C) 4.968

(D) 7.000

(E) 24.646

$\sqrt{\frac{3.2^2}{30} + \frac{3.8^2}{40}} = .838$

16.

$n = 15$

Starting time for hourly wage employees at a large manufacturing plant is 7 A.M. If an employee clocks in before 7:15 A.M., he is not marked as being late for work and his pay is not reduced. A random sample of 15 daily time sheets from the past two years showed that the average number of employees who arrived at work between 7 A.M. and 7:15 A.M. each day was 23 with a standard deviation of 6. Assume that the assumptions for inference have been met. Construct a 90% confidence interval for the mean number of employees who arrive at work during this time frame each day.

(A) 23 ± 2.728

(B) 23 ± 3.315

(C) 23 ± 2.630

(D) 23 ± 3.301

(E) 23 ± 2.894

$t^*_{14} = \text{invT}(.95, 14) = 1.7613$

$23 \pm 1.7613 \frac{6}{\sqrt{15}}$

23 ± 2.729

17.

A random sample of 32 games is chosen for a professional basketball team, team A, and their results are recorded. The team averaged 88 points per game with a standard deviation of 8. The same is done for a second team, team B, with this team averaging 90 points per game with a standard deviation of 6. A 95% confidence interval is constructed for the difference in points scored per game between the two teams. What do the results of the confidence interval show?

(A) We can be 95% confident that, on average, team A scores between 1.54 and 5.54 more points per game than team B.

(B) We can be 95% confident that, on average, team A scores between 1.54 and 5.54 fewer points per game than team B.

(C) We can be 95% confident that, on average, team A scores between 1.54 points fewer than and 5.54 points more than team B.

(D) We can be 95% confident that, on average, team A scores between 5.54 points fewer than and 1.54 points more than team B.

(E) The conditions necessary to find a 95% confidence interval have not been met.

$(-5.54, 1.54)$

18.
A company is interested in comparing the mean sales revenue per salesperson at two different locations. The manager takes a random sample of 10 salespersons from each location independently and records the sales revenue generated by each person during the last 4 weeks. He decides to use a t-test to compare the mean sales revenue at the two locations. Which of the following assumptions is necessary for the validity of the t-test?

~~(A)~~ The population standard deviations at both locations are equal.
~~(B)~~ The population standard deviations at both locations are not equal.
~~(C)~~ The population standard deviations at both locations are known.
(D) The population of the sales records at each location is normally distributed.
(E) The population of the difference in sales records computed by pairing one salesperson from each location is normally distributed.

19.
To work in the word-processing department at Dewey, Cheatem, and Howe, a large center-city law firm, you must be able to type at least 80 words per minute. The director of the human resources department is revising the job description for word processors. She believes that it is possible to adjust the typing speed upward and still have a sufficient number of qualified candidates. She takes a random sample of 15 employees from the word-processing department and gives them a typing test. The mean typing speed is 93 words per minute with a standard deviation of 7 words per minute. Assume that typing speeds follow an approximately normal distribution. A 98% confidence interval for the mean number of words typed by word processors at this law firm is (88.26, 97.74). What is the t^* critical value used to compute this interval?

(A) 2.131
(B) 2.145
(C) 2.249
(D) 2.264
(E) 2.624

$ME = 97.74 - 93 = 4.74$

$ME = t^* \frac{s}{\sqrt{n}}$

$4.74 = t^* \frac{7}{\sqrt{15}}$

$t^* = 2.623$

20.

Two professors, A and B, got into an argument about who grades tougher. Professor A insisted that his grades were lower than those for Professor B. In order to test this theory, each professor took a random sample of 25 student grades and conducted a test of significance. The graphical displays showed that each grade distribution was approximately normal. The results are recorded below.

H_o: Population mean of **Professor A** equals that of **Professor B**
H_a: Population mean of **Professor A** is less than that of **Professor B**

	Professor A	Professor B
Count:	25	25
Mean:	79	82
Std dev:	6	4
Std error:	1.2	0.8

Using unpooled variances

Student's t:	−2.08
DF:	41.8144
P-values:	0.022 $< \alpha = .05$

there is a difference.

Which of the following conclusions is/are supported by the results of the significance test?

~~I.~~ At the $\alpha = 0.05$ level, we have evidence to show that every student in Professor A's class scored lower than every student in Professor B's class.

This is true → ~~II.~~ If there were no difference in grades between the two professors, then we could get results as extreme as those from the samples approximately 2.2% of the time.

~~III.~~ The test results are not valid, since the conditions necessary to perform the test were not met.

They are met.

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

FREE RESPONSE

Questions 1-3

Spend about 45 minutes on this part of the exam.

1. 2004 Form B Question 4

The principal at Crest Middle School, which enrolls only sixth-grade students and seventh-grade students, is interested in determining how much time students at that school spend on homework each night. The table below shows the mean and standard deviation of the amount of time spent on homework each night (in minutes) for a random sample of 20 sixth-grade students and a separate random sample of 20 seventh-grade students at this school.

	Mean	Standard Deviation
Sixth-grade students	27.3	10.8
Seventh-grade students	47.0	12.4

Based on dotplots of these data, it is not unreasonable to assume that the distribution of times for each grade were approximately normally distributed.

(a) Estimate the difference in mean times spent on homework for all sixth- and seventh-grade students in this school using an interval. Be sure to interpret your interval.

Independent ✓
random ✓
independent groups ✓
10% condition ✓

2 Sample t interval

(−27.15, −12.25)

I am 95% confident that the difference between the mean times spent on homework for all 6th and 7th grade students in this school is between (−27.15 and −12.25.

(b) An assistant principal reasoned that a much narrower confidence interval could be obtained if the students were paired based on their responses; for example, pairing the sixth-grade student and the seventh-grade student with the highest number of minutes spent on homework, the sixth-grade student and seventh-grade student with the next highest number of minutes spent on homework, and so on. Is the assistant principal correct in thinking that matching students in this way and then computing a matched-pairs confidence interval for the mean difference in time spent on homework is a better procedure than the one used in part (a) ? Explain why or why not.

No, the students aren't naturally paired, they are independent.

2. 2006 Form B Question 4

The developers of a training program designed to improve manual dexterity claim that people who complete the 6-week program will increase their manual dexterity. A random sample of 12 people enrolled in the training program was selected. A measure of each person's dexterity on a scale from 1 (lowest) to 9 (highest) was recorded just before the start of and just after the completion of the 6-week program. The data are shown in the table below.

Person	Before Program	After Program
A	6.7	7.8
B	5.4	5.9
C	7.0	7.6
D	6.6	6.6
E	6.9	7.6
F	7.2	7.7
G	5.5	6.0
H	7.1	7.0
I	7.9	7.8
J	5.9	6.4
K	8.4	8.7
L	6.5	6.5
Total	81.1	85.6

Can one conclude that the mean manual dexterity for people who have completed the 6-week training program has significantly increased? Support your conclusion with appropriate statistical evidence.

$H_0: \mu_d = 0$

$H_A: \mu_d > 0$

$\alpha = .05$

paired t - test on the differences.

$t = 3.54$

$p = .0023$

independent ✓

random: No, but it is an experiment.

paired data ✓

10% condition ✓

Nearly normal ✓

With a p-value of .0023 I reject the null hypothesis. There is enough evidence to conclude that the mean manual dexterity for people has increased after the 6-week training program.