

Name: _____

Key

STATISTICS

PART 4 PRACTICE EXAM 1

Time – 1 hour and 30 minutes

Number of multiple choice questions – 20

Number of free response questions - 3

I am 95% confident that there are mistakes in this practice exam. ☺

1. A poll was conducted in the San Francisco Bay Area after the San Francisco Giants lost the World Series to the Anaheim Angels about whether the team should get rid of a pitcher who lost two games during the series. Five hundred twenty-five adults were interviewed by telephone, and 55% of those responding indicated that the Giants should get rid of the pitcher. It was reported that the survey had a margin of error of 3.5%. Which of the following best describes what is meant by a 3.5% margin of error?

- a. About 3.5% of the respondents were not Giants fans, and their opinions had to be discarded.
- ☒ b. It's likely that the true percentage that favor of getting rid of the pitcher is between 51.5% and 58.5%
- c. About 3.5% of those contacted refused to answer the question.
- d. About 3.5% of those contacted said that they had no opinion on the matter.
- e. About 3.5% thought their answer was in error and are likely to change their mind.

2. Two months before a statewide election, 532 respondents in a poll of 1000 randomly selected registered voters indicated that they favored Candidate A for governor ($\hat{p}_1 = 0.532$). One month before the election, a second poll of 900 registered voters was conducted and 444 respondents indicated that they favored Candidate A ($\hat{p}_2 = 0.493$). A 95% two-proportion z confidence interval for the true difference between the proportions favoring Candidate A in the first and second polls was constructed and found to be $(-0.0063, 0.0837)$. Which of the following is the best interpretation of this interval?

- ☒ a. There has not been a significant drop in support for Candidate A.
- b. There has been a significant drop in support for Candidate A.
- c. There has been no change in support for Candidate A.
- d. At the 5% level of significance, a test of $H_0: p_1 = p_2$ vs. $H_A: p_1 > p_2$ would yield exactly the same conclusion as the found confidence interval.
- e. Since support for Candidate A has fallen below 50%, she is unlikely to win a majority of votes in the general election.

3. A poll is taken to measure the proportion of voters who plan to vote for an ex-actor for Governor. A 95% confidence interval is constructed, based on a sample survey of prospective voters. The conditions needed to construct such an interval are present and the interval constructed is $(0.35, 0.42)$. Which of the following *best* describes how to interpret this interval?

- a. The probability is 0.95 that about 40% of the covers will vote for the ex-actor.
- b. The probability is 0.95 that between 35% and 42% of the population will vote for the ex-actor.
- c. At least 35%, but not more than 42%, of the voters will vote for the ex-actor.
- d. The sample result is likely to be in the interval $(0.35, 0.42)$.
- ☒ e. It is likely that the true proportion of covers who will cote for the ex-actor is between 35% and 42%.

4. Two sampling distributions of a sample mean for a random variable are to be constructed. The first (I) has a sample size of $n_1 = 8$ and the second (II) has a sample size of $n_2 = 35$. Which of the following statements is not true?

- a. Both sampling distributions I and II will have the same mean.
- b. Distribution I is more variable than Distribution II.
- c. The shape of Distribution I will be similar to the shape of the population from which it is drawn.
- ☒ d. The shape of each sampling distribution will be approximately normal.
- e. The shape of Distribution II will be approximately normal.

5. A researcher wants to determine if a newly developed anti-smoking program can be successful. At the beginning of the program, a sample of 1800 people who smoked at least 10 cigarettes a day were recruited for the study. These volunteers were randomly divided into two groups of 900 people. Each group received a set of anti-smoking materials and a lecture from a doctor and a cancer patient about the dangers of smoking. In addition, the treatment group received materials from the newly developed program. At the end of 2 months, 252 of the 900 people in the control group (the group that did not receive the new materials) reported that they no longer smoked. Out of the 900 people in the treatment group, 283 reported that they no longer smoked. Which of the following is an appropriate conclusion from this study?

- ☒ a. Because the P -value of this test is greater than $\alpha = 0.05$, we cannot conclude that the newly developed program is significantly different from the control program at reducing the rate of smoking.
- b. Since the proportion of people who have quit smoking in the experimental group is greater than the control group, we can conclude that the new program is effective at reducing the rate of smoking.
- c. Because the P -value of this test is less than $\alpha = 0.05$, we can conclude that the newly developed program is significantly different from the control program at reducing the rate of smoking.
- d. Because the difference in the proportions of those who have quit smoking in the control group (28%) and the experimental group (31.4%) is so small, we cannot conclude that there is a statistically significant difference between the two groups in terms of their rates of quitting smoking.
- e. The standard deviation of the difference between the two sample proportions is about 0.22. This is so small as to give us good evidence that the new program is more effective at reducing the rate of smoking.

6. A wine maker advertises that the mean alcohol content of the wine produced by his winery is 11%. A 95% confidence interval, based on a random sample of 100 bottles of wine yields a confidence interval for the true alcohol content of (10.5, 10.9). Could this interval be used as part of a hypothesis test of the null hypothesis $H_0: p = 0.11$ versus the alternative hypothesis $H_A: p \neq 0.11$ at the 0.05 level of confidence?

- a. No, you cannot use a confidence interval in a hypothesis test.
- ☒ b. Yes, because 0.11 is not contained in the 95% confidence interval, a two-sided test at the 0.05 level of significance would provide good evidence that the true mean content is different from 11%.
- c. No, because we do not know that the distribution is approximately normally distributed.
- d. Yes, because 0.11 is not contained in the 95% confidence interval, a two-sided test at the 0.05 level of significance would fail to reject the null hypothesis.
- e. No, confidence intervals can only be used in one-sided significance tests.

7. A national polling organization wishes to generate a 98% confidence interval for the proportion of voters who will vote for candidate Iam Sleazy in the next election. The poll is to have a margin of error no more than 3%. What is the minimum sample size needed for this interval?

- a. 6032
- b. 1508
- c. 39
- d. 6033
- ☒ e. 1509

8. In a test of the hypothesis $H_0: p = 0.7$ against $H_A: p > 0.7$ the power of the test when $p = 0.8$ would be greatest for which of the following?

- ☒ a. $n = 30, \alpha = 0.10$
- b. $n = 30, \alpha = 0.05$
- c. $n = 25, \alpha = 0.10$
- d. $n = 25, \alpha = 0.05$
- e. It cannot be determined from the information given.

9. A statistics class wanted to construct a 90% confidence interval for the difference in the number of advanced degrees held by male and female faculty members at their school. They collected degree data from all the male and female faculty members and then used these data to construct the desired 90% confidence interval. Is this an appropriate way to construct a confidence interval?

- a. No, because we don't know that the distributions involved are approximately normal.
- b. Yes, but only if the number of men and the number of women are equal because our calculations will be based on different scores.
- c. Yes, but only if the distribution of difference scores has no outliers or extreme skewness.
- ☒ d. No, because all the data were available, there is no need to construct a confidence interval for the true difference between the number of degrees.
- e. No, confidence intervals can only be constructed on independent samples, not on paired differences.

10. A researcher conducts a study of the effectiveness of a relaxation technique designed to improve the length of time a scuba diver can stay at a depth of 60 feet with a 80 cu. Ft. tank of compressed air. The average bottom time for a group of divers before implementation of the program was 48 minutes and the average bottom time after implementation of the program was 54 minutes with a P -value of 0.24. Which of the following is the best interpretation of this finding?

- a. There is a 2.4% chance that the new technique is effective at increasing bottom time.
- ☒ b. If the new technique was not effective, there is only a 2.4% chance of getting 54 minutes or more by chance alone.
- c. 97.6% of the divers in the study increased their bottom times.
- d. We can be 97.6% confident that the new technique is effective at increasing bottom time.
- e. The new technique does not appear to be effective at increasing bottom time.

11.

Two random samples of American adults are taken, and the religious affiliations of the individuals involved are recorded. In the first sample of 200 adults, 66 of the individuals are Christians. In the second sample of 140 adults, 12 of the individuals are Buddhists. Assume the two samples are independent. Which of the following should be used to construct a 95% confidence interval for the difference in proportions for adult Americans who practice the two religions?

- (A) $0.0786 \pm 1.96\sqrt{0.0005}$
- ☒ (B) $0.2443 \pm 1.96\sqrt{0.0017}$
- (C) $0.33 \pm 1.96\sqrt{0.0011}$
- (D) The conditions necessary for computing a confidence interval have not been met; therefore, a confidence interval should not be computed.
- (E) Because the sample sizes for the two proportions are not equal, a confidence interval cannot be computed.

12.

Based upon a random sample of 30 seniors in a high school, a guidance counselor finds that 20 of these seniors plan to attend an institution of higher learning. A 90% confidence interval constructed from this information yields (0.5251, 0.80823). Which of the following is a correct interpretation for this interval?

- (A) We can be 90% confident that 52.51% to 80.82% of our sample seniors plan to attend an institution of higher learning.
- ☒ (B) We can be 90% confident that 52.51% to 80.82% of seniors at this high school plan to attend an institution of higher learning.
- (C) We can be 90% confident that 52.51% to 80.82% of seniors in any school plan to attend an institution of higher learning.
- (D) This interval will capture the true proportion of seniors from this high school who plan to attend an institution of higher learning 90% of the time.
- (E) This interval will capture the proportion of seniors in our sample who plan to attend an institution of higher learning 90% of the time.

13.

In general, how does quadrupling the sample size affect the width of a confidence interval?

- ☒ (A) The width of the interval becomes four times as large.
- (B) The width of the interval becomes two times as large.
- ☒ (C) The width of the interval becomes half as large.
- (D) The width of the interval becomes one-quarter as large.
- (E) We need to know the sample size to be able to determine the effect.

14.

A no-appointment haircutter advertises an average wait time of 15 minutes for customers. A consumer advocacy group has received several complaints from customers who believe the wait time is really 30 minutes. The advocacy group randomly selects 30 customers, records wait times, and calculates the power of the test to be 50%. In order to increase the power of the test *as much as possible*, the advocacy group should

- ☒ (A) increase the sample size and increase the value of α .
- ☐ (B) increase the sample size and decrease the value of α .
- ☐ (C) increase the sample size but keep the same value for α .
- ☐ (D) decrease the sample size and increase the value of α .
- ☐ (E) decrease the sample size and decrease the value of α .

15.

A magazine claims that 25.1% of all women enjoy gardening. A researcher believes the percentage is higher and performs a test of $H_0: p = 0.251$ versus $H_a: p > 0.251$. A random sample of 100 women yields significant results at the $\alpha = 0.05$ level. Which of the following statements about the confidence interval used to estimate the true population proportion of women who enjoy gardening must be true?

- ☐ (A) A 90% confidence interval contains the proportion 0.251.
- ☐ (B) A 90% confidence interval does not contain the proportion 0.251, because the value of 0.251 is above the upper limit of the interval.
- ☒ (C) A 90% confidence interval does not contain the proportion 0.251, because the value of 0.251 is below the lower limit of the interval.
- ☐ (D) A 95% confidence interval does not contain the proportion 0.251, because the value of 0.251 is above the upper limit of the interval.
- ☐ (E) A 95% confidence interval does not contain the proportion 0.251, because the value of 0.251 is below the lower limit of the interval.

16.

The p -value for a significance test is 0.0358. A correct interpretation of this p -value would be:

- ☐ (A) About 3.6% of the data are significant.
- ☐ (B) About 3.6% of all samples are significant.
- ☒ (C) About 3.6% of all samples would produce a test statistic at least as extreme as ours if the null hypothesis is true.
- ☐ (D) There is sufficient evidence to reject the null hypothesis.
- ☐ (E) There is sufficient evidence to fail to reject the null hypothesis.

17.

Randomly selected individuals were asked about their physical activity. Of 75 randomly selected men, 30 had walked for exercise in the preceding two weeks. Of 75 randomly selected women, 36 had walked for exercise in the preceding two weeks. Assume independence between the samples. Is there evidence to show a significant difference in the proportion of men and the proportion of women who walk for exercise?

- (A) Because the proportions are different, there is evidence to show a significant difference in the proportions of men and women who walk for exercise.
- (B) With $p = 0.162$, there is insufficient evidence to show a significant difference in the proportions of men and women who walk for exercise.
- ☒ (C) With $p = 0.324$, there is insufficient evidence to show a significant difference in the proportions of men and women who walk for exercise.
- (D) With $p = 0.838$, there is insufficient evidence to show a significant difference in the proportions of men and women who walk for exercise.
- (E) The conditions necessary to perform a significance test have not been met; therefore, a conclusion cannot be drawn.

18.

A congressman mails a questionnaire to his constituents asking if the United States should use military force to overthrow violent dictators in controversial areas of the world. Of the 500 people who respond, 35% believe the United States should use military force in this situation. On a talk show, the politician claims that only 35% of his constituents (with a 4% margin of error) believe in using military force. Which assumption for constructing a confidence interval is violated?

- (A) The population is ten times as large as the sample.
- ☒ (B) The data constitute a simple random sample from the population of interest.
- (C) The count of successes, $n\hat{p}$, is 10 or more.
- (D) The count of failures, $n(1 - \hat{p})$, is 10 or more.
- (E) There are no violations for constructing a confidence interval.

19.

Owners of a day-care chain wish to determine the proportion of families in need of day care for the town of Bockville. Bockville is estimated to have 1000 families. The owners of the day-care chain randomly sample 50 families and find that 60% of them have a need for day-care services. Which of the following is a condition necessary for constructing a confidence interval for a **proportion** that has *not* been met?

- (A) The data constitute a representative random sample from the population of interest.
- (B) The sample size is less than 10% of the population size.
- (C) The counts of those who need day care and those who don't need day care are 10 or more.
- (D) The distribution of sample values is approximately normally distributed.
- (E) All conditions necessary for constructing a confidence interval for the proportion seem to be met.

20.

In a very large school district, the food services administrator wishes to determine the proportion of students who will buy a school lunch to within ± 0.03 . Using the most conservative estimate for p , how many students should this administrator survey to have 90% confidence?

- (A) 164
- (B) 271
- (C) 457
- (D) 752
- (E) 1844

FREE RESPONSE

Questions 1-3

Spend about 45 minutes on this part of the exam.

1. 2010a #3

A humane society wanted to estimate with 95 percent confidence the proportion of households in its county that own at least one dog.

(a) Interpret the 95 percent confidence level in this context.

The humane society selected a random sample of households in its county and used the sample to estimate the proportion of all households that own at least one dog. The conditions for calculating a 95 percent confidence interval for the proportion of households in this county that own at least one dog were checked and verified, and the resulting confidence interval was 0.417 ± 0.119 .

(b) A national pet products association claimed that 39 percent of all American households owned at least one dog. Does the humane society's interval estimate provide evidence that the proportion of dog owners in its county is different from the claimed national proportion? Explain.

(c) How many households were selected in the humane society's sample? Show how you obtained your answer.

2. 2006b #2

A large company has two shifts—a day shift and a night shift. Parts produced by the two shifts must meet the same specifications. The manager of the company believes that there is a difference in the proportions of parts produced within specifications by the two shifts. To investigate this belief, random samples of parts that were produced on each of these shifts were selected. For the day shift, 188 of its 200 selected parts met specifications. For the night shift, 180 of its 200 selected parts met specifications.

- (a) Use a 96 percent confidence interval to estimate the difference in the proportions of parts produced within specifications by the two shifts.

- (b) Based only on this confidence interval, do you think that the difference in the proportions of parts produced within specifications by the two shifts is significantly different from 0? Justify your answer.

3. 2005a #4

Some boxes of a certain brand of breakfast cereal include a voucher for a free video rental inside the box. The company that makes the cereal claims that a voucher can be found in 20 percent of the boxes. However, based on their experiences eating this cereal at home, a group of students believes that the proportion of boxes with vouchers is less than 0.2. This group of students purchased 65 boxes of the cereal to investigate the company's claim. The students found a total of 11 vouchers for free video rentals in the 65 boxes.

Suppose it is reasonable to assume that the 65 boxes purchased by the students are a random sample of all boxes of this cereal. Based on this sample, is there support for the students' belief that the proportion of boxes with vouchers is less than 0.2 ? Provide statistical evidence to support your answer.

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Question 3

Intent of Question

The primary goals of this question were to assess students' ability to (1) interpret the meaning of a confidence level; (2) use a confidence interval to test the plausibility of a claim about the value of a population parameter; (3) perform a sample size calculation related to a confidence interval.

Solution

Part (a):

The 95 percent confidence level means that if one were to repeatedly take random samples of the same size from the population and construct a 95 percent confidence interval from each sample, then in the long run 95 percent of those intervals would succeed in capturing the actual value of the population proportion of households in the county that own at least one dog.

Part (b):

No. The 95 percent confidence interval 0.417 ± 0.119 is the interval $(0.298, 0.536)$. This interval includes the value 0.39 as a plausible value for the population proportion of households in the county that own at least one dog. Therefore, the confidence interval does not provide evidence that the proportion of dog owners in this county is different from the claimed national proportion.

Part (c):

The sample proportion is 0.417, and the margin of error is 0.119. Determining the sample size requires solving the equation $0.119 = 1.96 \times \sqrt{\frac{0.417 \times (1 - 0.417)}{n}}$ for n .

Thus, $n = \frac{1.96^2 \times 0.417 \times (1 - 0.417)}{0.119^2} \approx 65.95$, so the humane society must have selected 66 households for its sample.

Scoring

Parts (a), (b) and (c) are scored as essentially correct (E), partially correct (P) or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the student provides a correct interpretation of the confidence level in the context of the study. A correct interpretation can take one of two approaches:

1. Based on the concept of repeated sampling, the response must fulfill the following three requirements:
 - Mentions repeated sampling or "in the long run" or "using this method"
 - Mentions that 95 percent of the intervals will capture the population proportion
 - Includes the context of the study

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Question 3 (continued)

2. Based on probability, the response must state that there is a 0.95 probability that a random sample *selected in the future* will produce an interval that captures the actual value of the population proportion of households in the county that have at least one dog.

Partially correct (P) if the student provides an interpretation of the confidence level that includes two of the three components required for the repeated sampling interpretation *OR* provides a correct probability interpretation, but not in context.

Incorrect (I) if the student attempts to interpret a particular confidence *interval* rather than the confidence *level* (for example, by saying that we are 95 percent confident that an interval that has been obtained includes the population proportion of households in the county that have at least one dog) *OR* provides an interpretation of the confidence level that mentions at most one of the three components required for the repeated sampling interpretation.

Part (b) is scored as follows:

Essentially correct (E) if the student correctly states that because 0.39 (or “the claimed value”) is in the computed interval, the interval does not provide evidence that the proportion of dog owners in the county is different from the claimed national proportion.

Partially correct (P) if the student indicates that the goal is to check whether the claimed value of 0.39 is in the computed interval but makes errors in implementation. Examples of errors include the following:

- The student notes that 0.39 is within the interval but does not draw a correct conclusion.
- The student makes an arithmetic error in computing the endpoints of the interval, but the conclusion is consistent with the computed interval.

OR

The student correctly notes that 0.39 is in the interval and then concludes that 0.39 is the population proportion for the county.

Incorrect (I) if the student does not recognize how to check whether the claim is consistent with the confidence interval.

Part (c) is scored as follows:

Essentially correct (E) if the student provides a correct equation with correct numerical values substituted, as well as a correct integer solution.

Partially correct (P) if the student provides a correct equation (and substitutions) but makes one or more of the following errors:

- Does not complete the calculation or completes the calculation incorrectly
- Uses 0.5 or 0.39 rather than 0.417 as the sample proportion
- Uses an incorrect but plausible z^* value
- Reports the answer as a non-integer value
- Gives the calculated value of n as a lower bound for the required sample size

Incorrect (I) otherwise.

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Question 3 (continued)

Notes

- It is acceptable to use $z^* = 2$ instead of 1.96.
- It is acceptable for the response to round up or down to get an integer answer.

4 Complete Response

All three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and one part incorrect

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct and two parts incorrect

OR

Two parts partially correct and one part incorrect

PT 1 #2

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2006 SCORING GUIDELINES (Form B)

Question 2

Intent of Question

The primary goals of this question are to evaluate a student's ability to: (1) identify and compute an appropriate confidence interval, after checking the necessary conditions; (2) interpret the interval in the context of the question; and (3) use the confidence interval to conduct an appropriate test of significance.

Solution

Part (a):

Step 1: Identifies the appropriate confidence interval by name or formula and checks appropriate conditions.

Two sample z confidence interval for $p_D - p_N$, the difference in the proportions of parts meeting specifications for the two shifts OR $(\hat{p}_D - \hat{p}_N) \pm z^* \sqrt{\frac{\hat{p}_D(1 - \hat{p}_D)}{n_D} + \frac{\hat{p}_N(1 - \hat{p}_N)}{n_N}}$.

- Conditions:
1. Independent random samples from two separate populations
 2. Large samples, so normal approximation can be used

The problem states that random samples of parts were selected from the two different shifts. We need to assume that these parts were produced independently. That is, each employee works the day shift or night shift, but not both, and the machine quality does not vary over time. Since the sample sizes are both 200 and the number of successes (188 and 180) and the number of failures (12 and 20) for each shift are larger than 10, it is reasonable to use the large sample procedures.

Step 2: Correct mechanics

$$\begin{aligned}\hat{p}_D &= \frac{188}{200} = 0.94 \text{ and } \hat{p}_N = \frac{180}{200} = 0.90 \\ (0.94 - 0.9) &\pm 2.0537 \sqrt{\frac{0.94 \times 0.06}{200} + \frac{0.9 \times 0.1}{200}} \\ 0.04 &\pm 2.0537 \times 0.0271 \\ 0.04 &\pm 0.0556 \\ (-0.0156, &0.0956)\end{aligned}$$

Step 3: Interpretation

Based on these samples, we can be 96 percent confident that the difference in the proportions of parts meeting specifications for the two shifts is between -0.0156 and 0.0956 .

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Question 2 (continued)

Part (b):

Since zero is in the 96 percent confidence interval, zero is a plausible value for the difference $p_D - p_N$, and we do not have evidence to support the manager's belief. In other words, there does not appear to be a statistically significant difference between the proportions of parts meeting specifications for the two shifts at the $\alpha = 0.04$ level.

Scoring

Part (a) is scored according to the number of correct steps. Each of the first three steps is scored as essentially correct (E) or incorrect (I). Part (b) is scored as essentially correct (E) or incorrect (I).

Notes for Step 1:

The student must identify an appropriate confidence interval and comment on both independence and large sample sizes in order to get this step essentially correct.

Minimum amount of information on independence and large sample sizes needed for an essentially correct response: independence with a check mark AND an indication that the number of successes and the number of failures is larger than 10 (or larger than 5) for both samples.

The student does not need to restate the fact that these are random samples.

Notes for Step 2: An identifiable minor arithmetic error in Step 2 will not necessarily change a score from essentially correct to incorrect.

Alternative Solutions for Step 2	
Procedure	96% Confidence Interval
Calculator	$(-0.0155652, 0.0955652)$
Wilson Estimator	$(-0.0169858, 0.0961937)$

Part (b) is essentially correct (E) if the student comments on the fact that zero is contained in the confidence interval and the justification links this outcome to a 96 percent confidence level, or a 0.04 significance level, and includes a statement indicating that the data do not support the manager's belief that there is a difference in the proportion of parts that meet specifications produced by the two shifts.

Part (b) is incorrect (I) if the student says no because zero is in the confidence interval OR simply says no without providing relevant justification.

Note: If a 95 percent confidence interval is used, then the maximum score is 3.

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Question 2 (continued)

4 Complete Response

All three steps of the confidence interval in part (a) are essentially correct, and part (b) is essentially correct.

3 Substantial Response

All three steps of the confidence interval in part (a) are essentially correct, and part (b) is incorrect.

OR

Two steps of the confidence interval in part (a) are essentially correct and part (b) is essentially correct.

2 Developing Response

Two steps of the confidence interval in part (a) are essentially correct, and part (b) is incorrect.

OR

One step of the confidence interval in part (a) is essentially correct, and part (b) is essentially correct.

1 Minimal Response

One step of the confidence interval in part (a) is essentially correct, and part (b) is incorrect.

OR

Part (b) is essentially correct.

Question 4

Solution

This question is divided into four parts.

Part (a): State a correct pair of hypotheses.

Let p = the proportion of boxes of this brand of breakfast cereal that include a voucher for a free video rental.

$$H_0 : p = 0.2$$

$$H_a : p < 0.2$$

Part (b): Identify a correct test (by name or by formula) and check appropriate conditions.

One-sample z -test for a proportion OR
$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$

Conditions:

1. $np_0 = 65 \times 0.2 = 13 > 10$ and $n(1 - p_0) = 65 \times 0.8 = 52 > 10$.
2. It is reasonable to assume that the company produces more than $65 \times 10 = 650$ boxes of this cereal ($N > 10n$).
3. The observations are independent because it is reasonable to assume that the 65 boxes are a random sample of all boxes of this cereal.

Part (c): Use correct mechanics and calculations, and provide the p -value (or rejection region).

The sample proportion is $\hat{p} = \frac{11}{65} = 0.169$. The test statistic is $z = \frac{0.169 - 0.2}{\sqrt{\frac{0.2(1 - 0.2)}{65}}} = -0.62$ and the p -value is

$$P(Z < -0.62) = 0.2676.$$

Part (d): State a correct conclusion, using the result of the statistical test, in the context of the problem.

Since the p -value = 0.2676 is larger than any reasonable significance level (e.g., $\alpha = 0.05$), we cannot reject the company's claim. That is, we do not have statistically significant evidence to support the student's belief that the proportion of cereal boxes with vouchers is less than 20 percent.

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2005 SCORING GUIDELINES

Question 4 (continued)

Scoring

The question is divided into four parts. Each part is scored as essentially correct (E) or incorrect (I).

Part (a) is essentially correct (E) if the student states a correct pair of hypotheses.

Notes:

1. Since the proportion was defined in the stem, standard notation for the proportion (p or π) need not be defined in the hypotheses.
2. Nonstandard notation must be defined correctly.
3. A two-sided alternative is incorrect for this part.

Part (b) is essentially correct (E) if the student identifies a correct test (by name or by formula) and checks for appropriate conditions.

Notes:

1. $np_0 > 5$ and $n(1 - p_0) > 5$ are OK as long as appropriate values are used for n and p_0 .
2. Since students cannot check the actual population size, they do not need to mention it.
3. The stem of the problem indicates this is a random sample so it (or a discussion of independence) does not need to be repeated in the solution.

Part (c) is essentially correct (E) if no more than one of the following errors is present in the student's work:

- Undefined, nonstandard notation is used; OR
- The correct z -value = -0.62 is given with no setup for the calculation; OR
- The incorrect z -value = -0.67 is given because \hat{p} was used in the calculation of the standard error. For this incorrect z -value, the p -value = 0.2514 . ; OR
- The incorrect z -value is calculated because of a minor arithmetic error.

Part (c) is incorrect (I) if:

- Inference for a lower tail alternative is based on either two-tails p -value = 0.535 or the upper tail p -value = 0.734 ; OR
- An unsupported z -value other than -0.62 or -0.67 is given; OR
- The correct z -value = -0.62 is given but equated to an incorrect formula.

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Question 4 (continued)

Notes:

1. Students using a rejection region approach should have critical values appropriate for a lower tail test, e.g., for $\alpha = 0.05$ the rejection region is $z < -1.645$.
2. Other possible correct mechanics include:
 - Exact Binomial
 $X \sim \text{Binomial}(n=65, p=0.2)$. The exact p -value is $P(X \leq 11) = 0.33$.
 - Normal Approximation to Binomial (with or without a continuity correction)
 X is approximately Normal(13, 3.225). The approximate p -value using the continuity correction is $P\left(Z \leq \frac{11 + 0.5 - 13}{3.225}\right) = P(Z \leq -0.4651) = 0.3209$.
 - Confidence interval approach – provided there is a reasonable interpretation tied to a significance level. For example if $\alpha = 0.05$, and $p = 0.20$ is within a 95% upper confidence bound (0, 0.2457) or a two-tailed 90% confidence interval (0.0927, 0.2457).

Part (d) is essentially correct (E) if the student states a correct conclusion in the context of the problem, using the result of the statistical test.

Notes:

1. If both an α and a p -value (or critical value) are given, the linkage is implied.
2. If no α is given, the solution must be explicit about the linkage by giving a correct interpretation of the p -value or explaining how the conclusion follows from the p -value.
3. If the p -value in part (c) is incorrect but the conclusion is consistent with the computed p -value, part (d) can be considered as essentially correct (E).
4. If a student accepts the null hypothesis and concludes the proportion really is 0.20, this part is incorrect (I).

Each essentially correct (E) response counts as 1 point, each partially correct (P) response counts as ½ point.

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|----------|-----------------------------|
| 4 | Complete Response |
| 3 | Substantial Response |
| 2 | Developing Response |
| 1 | Minimal Response |

Note: If a response is in between two scores (for example, 2 ½ points), use a holistic approach to determine whether to score up or down depending on the strength of the response and communication.