Name: Key

STATISTICS

PART 4 PRACTICE EXAM 3

Time – 1 hour and 30 minutes

Number of multiple choice questions – 20

Number of free response questions – 3

1.

Two hundred students were classified by gender and hostility level (low, medium, high), as measured by an HLT-test. The results were the following:

	Hostility Level		
	Low	Medium	High
Male	35	40	5
Female	62	50	8

If the hostility level among students were independent of their gender, then how many female students would we expect to show the medium HLT score?

(A) 25
(B) 45
(C) 54
(D) 60
(E) 75

2.

In the jury pool available for this week, 30 percent of potential jurors are women. If a jury of 12 is to be selected at random, what is the expected number of men in the group?

(A) 12(0.30)
(B) 12(0.50)
(C) 12(0.70)
(D) 12(0.30)(0.70)
(E) $\sqrt{12(0.30)(0.70)}$

3.

A medicine is known to produce side effects in 1 in 5 patients taking it. Suppose a doctor prescribes the medicine to 4 unrelated patients. What is the probability that none of the patients will develop side effects?

(A) 0.8000
(B) 0.4096
(C) 0.2500
(D) 0.2000
(E) 0.0016

4.

An automobile service station performs only oil changes and tire replacements. Eighty percent of its customers request an oil change. Of those who request an oil change, only 20 percent request a tire replacement. What is the probability that the next customer will request both an oil change and a tire replacement?

(A) 0.16
(B) 0.20
(C) 0.25
(D) 0.80
(E) 0.85

5.

Which of the following is a discrete random variable?

(A) The number of times a student guesses the answers to questions on a certain test
(B) The amount of gasoline purchased by a customer
(C) The amount of mercury found in fish caught in the Gulf of Mexico
(D) The height of water-oak trees
(E) The time elapsed until the first field goal at home football games

6.

In which of the following situations is a binomial not an appropriate model to describe the outcome?

(A) The number of heads in three tosses of a coin
(B) The number of rainy days in a given week
(C) The number of girls in a family of 5 children
(D) The number of students present in a class of 22
(E) The number of defective computer monitors out of 7 purchased

7.

Which of the following statements about any two events A and B is true?

(A) $P(A \cup B)$ implies events A and B are independent.
(B) $P(A \cup B) = 1$ implies events A and B are mutually exclusive.
(C) $P(A \cap B) = 0$ implies events A and B are independent.
(D) $P(A \cap B) = 0$ implies events A and B are mutually exclusive.
(E) $P(A \cap B) = P(A) - P(B)$ implies A and B are equally likely events.

7.

Let X be a random variable that follows a t-distribution with a mean of 75 and a standard deviation of 8. Which of the following is (are) equivalent to $P(X > 85)$?

I. $P(X < 65)$
II. $P(X \geq 65)$
III. $1 - P(X < 65)$

a. I only
b. II only
c. III only
d. I and III only
e. I, II, and III

8.

Given $P(A) = 0.4$, $P(B) = 0.3$, $P(B|A) = 0.2$. What are $P(A \text{ and } B)$ and $P(A \text{ or } B)$?

a. $P(A \text{ and } B) = 0.12$, $P(A \text{ or } B) = 0.58$
b. $P(A \text{ and } B) = 0.08$, $P(A \text{ or } B) = 0.62$
c. $P(A \text{ and } B) = 0.12$, $P(A \text{ or } B) = 0.62$
d. $P(A \text{ and } B) = 0.08$, $P(A \text{ or } B) = 0.58$
e. $P(A \text{ and } B) = 0.08$, $P(A \text{ or } B) = 0.70$

Use this information for problems 10 and 11

At a local community college, 90% of students take English. 80% of those who don't take English take art courses, while only 50% of those who do take English take art.

10.

What is the probability that a student takes art?

a. 0.80
b. 0.53
c. 0.50
d. 1.3
e. 0.45

11.

What is the probability that a student who takes art doesn't take English?

a. 0.08
b. 0.10
c. 0.8
d. 0.85
e. 0.15

12.

Tom's career batting average is 0.265 with a standard deviation of 0.035. Larry's career batting average is 0.283 with a standard deviation of 0.029. The distribution of both averages is approximately normal. They play for different teams and there is reason to believe that their career averages are independent of each other. For any given year, what is the probability that Tom will have a higher batting average than Larry?

a. 0.389
b. 0.345
c. 0.589
d. 0.655
e. You cannot answer this question since the distribution for the difference between their averages cannot be determined from the data given.

13.

Which of the following statements is (are) correct?

I. The area under a probability density curve for a continuous random variable is 1.
II. A random variable is a numerical outcome of a random event.
III. The sum of the probabilities for a discrete random variable is 1.

a. II only
b. I and II
c. I and III
d. II and III
e. I, II, and III

14.

Given $P(A) = 0.60$, $P(B) = 0.30$, and $P(A|B) = 0.50$. Find $P(A \cup B)$.

a. 0.90
b. 0.18
c. 0.40
d. 0.72
e. 0.75

15.

Let X be the number of points awarded for winning a game that has the following probability distribution:

X	0	2	3
P(X)	0.2	0.5	0.3

Let Y be the random variable whose sum is the number of points that results from two independent repetitions of the game. Which of the following is the probability distribution for Y?

a.

Y	0	2	3
P(Y)	0.2	0.5	0.3

b.

Y	0	4	6
P(Y)	0.2	0.5	0.3

c.

Y	0	2	3	4	6
P(Y)	0.2	0.25	0.15	0.25	0.15

d.

Y	0	2	3	4	5	6
P(Y)	0.04	0.2	0.12	0.25	0.3	0.09

e.

Y	0	2	3	4	5	6
P(Y)	0.04	0.10	0.06	0.25	0.15	0.09

16.

You play a game that involves rolling a die. You either win or lose \$1 depending on what number comes up on the die. If the number is even, you lose \$1, and if it is odd, you win \$1. However, the die is weighted and has the following probability distribution for the various faces:

Face	**1**	**2**	**3**	**4**	**5**	**6**
Win (x)	+1	−1	+1	−1	+1	−1
$p(x)$	0.15	0.20	0.20	0.25	0.1	0.1

Given that you win rather than lose, what is the probability that you rolled a "5"?

a. 0.50
b. 0.10
c. 0.45
d. 0.22
e. 0.55

17.

It is the morning of the day that Willie and Baxter have planned their long-anticipated picnic. Willie reads, with some distress, that there is a 65% probability of rain in their area today. Which of the following best describes the most likely way that probability was arrived at?

a. It rains 65% of the time on this date each year.
b. Historically, in the United States, it has rained 65% of the time on days with similar meteorological conditions as today.
c. Historically, it rains 65% of the days during this month.
d. Historically, in this area, it has rained 65% of the time on days with similar meteorological conditions as today.
e. This is the result of a simulation conducted by the weather bureaus.

Use the following information for questions 18 and 19

Baxter is a 60% free-throw shooter who gets fouled during a game and gets to shoot what is called a "one-and-one" (that is, he gets to take a second shot—a bonus—if and only if he makes his first shot; each free throw, if made, is worth one point). Baxter can make 0 points (because he misses his first shot), 1 point (he makes the first shot, but misses the bonus), or 2 points (he makes his first shot and the bonus).

18.

Assuming that each shot is independent, how many points is Baxter *most likely* to make in a one-and-one situation?

a. 2
b. 1
c. 0
d. 0.96
e. None of these is correct.

19.

Assuming that each shot is independent, how many points will Baxter make *on average* in a one-and-one situation?

a. 2
b. 0.96
c. 0
d. 1
e. 0.36

20.

A fair die is to be rolled 8 times. What is the probability of getting at least one 4?

a. $\frac{1}{6}$

b. $\binom{8}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^7$

c. $1-\binom{8}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^7$

d. $\binom{8}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^6+\binom{8}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^5+\cdots+\binom{8}{8}\left(\frac{1}{6}\right)^8\left(\frac{5}{6}\right)^0$

e. $1-\binom{8}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^8$

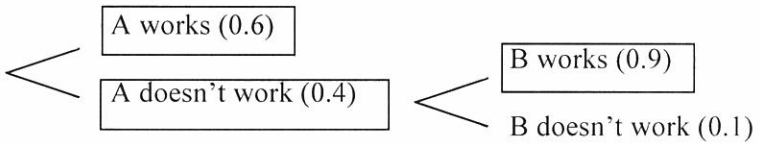
Question 4 (cont'd.)

Scoring

Each part is scored as essentially correct, partially correct, or incorrect.

Part (a) is essentially correct if the probabilities of cure are calculated correctly with justification for both plans.

Plan I:



$P(\text{Cure}_{\text{I}}) = 0.6 + (0.4 \times 0.9) = 0.96$

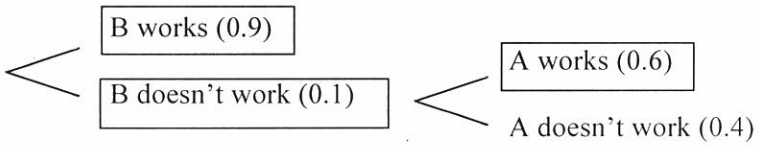
OR

$P(\text{Cure}_{\text{I}}) = P(\text{A} \cup \text{B}) = 0.6 + 0.9 - (0.6 \times 0.9) = 0.96$

OR

$P(\text{Cure}_{\text{I}}) = 1 - P(\text{not A})P(\text{not B}) = 1 - (0.4) \times (0.1) = 0.96$

Plan II:



$P(\text{Cure}_{\text{II}}) = 0.9 + (0.1 \times 0.6) = 0.96$

OR

$P(\text{Cure}_{\text{II}}) = P(\text{B} \cup \text{A}) = 0.9 + 0.6 - (0.9 \times 0.6) = 0.96$

OR

$P(\text{Cure}_{\text{II}}) = 1 - P(\text{not B})P(\text{not A}) = 1 - (0.1) \times (0.4) = 0.96$

Question 4 (cont'd.)

Part (a) is partially correct if

one of the two probabilities is calculated correctly with justification,

OR

both probabilities are correct with incomplete justifications.

Part (b) is essentially correct if the expected costs per child are calculated correctly with justification for both plans.

The expected cost per child when Plan I is used for treatment is:

$$E(\text{Cost}_{\text{I}}) = \$50 \times 0.6 + \$130 \times 0.4 = \$30 + \$52 = \$82$$

OR

$$E(\text{Cost}_{\text{I}}) = \$50 + 0.4 \times \$80 = \$50 + \$32 = \$82$$

The expected cost per child when Plan II is used for treatment is:

$$E(\text{Cost}_{\text{II}}) = \$80 \times 0.9 + \$130 \times 0.1 = \$72 + \$13 = \$85$$

OR

$$E(\text{Cost}_{\text{II}}) = \$80 + 0.1 \times \$50 = \$80 + \$5 = \$85$$

Part (b) is partially correct if
the expected cost per child is calculated correctly with justification for one of the two plans,

OR

both expected costs are correct with incomplete justifications,

OR

the expected costs are incorrectly calculated but the probabilities involved add up to 1. For example the following computations would receive a partial.

The expected cost per child when Plan I is used for treatment is:

$$= \$50 \times 0.6 + \$80 \times 0.4 = \$30 + \$32 = \$62$$

The expected cost per child when Plan II is used for treatment is:

$$= \$80 \times 0.9 + \$50 \times 0.1 = \$72 + \$5 = \$77$$

Question 4 (cont'd.)

In contrast, the following computations would receive an incorrect because the probabilities involved do not add up to 1.

The expected cost per child when Plan I is used for treatment is:

$$= \$50 \times 0.6 + \$130 \times 0.36 = \$30 + \$46.80 = \$76.80$$

The expected cost per child when Plan II is used for treatment is:

$$= \$80 \times 0.9 + \$130 \times 0.06 = \$72 + \$7.80 = \$79.80$$

Part (c) is essentially correct if the recommendation contains a statistical argument based on parts (a) and (b). That is, the student must base the recommendation on probabilities from part (a) and expected values from part (b). The following two examples are essentially correct:

Since the probability that a child will be cured is the same under either plan, some other criterion must be used to make a recommendation. From a financial point of view, Plan I should be recommended because the expected cost per child is less than Plan II.

Since the probability that a child will be cured is the same under either plan, some other criterion must be used to make a recommendation. Parents might prefer Plan II, regardless of its higher cost, because their child is more likely to need only the first drug.

Part (c) is partially correct if the recommendation contains a statistical argument based only on part (a) or (b) but not both.

Part (c) is incorrect if no recommendation is made.

Question 4 (cont'd.)

4 Complete Response

All three parts essentially correct

3 Substantial Response

Two parts essentially correct and 1 part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and 2 parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct and either 0 or 1 part partially correct

OR

No parts essentially correct and 2 parts partially correct

#2

AP® STATISTICS
2003 SCORING GUIDELINES (Form B)

Question 2

Solution

Part (a): $P(age\ 31-45)=\frac{89}{207}=0.42995$

Part (b): $P(age\ 31-45 \mid income\ over\ 50,000)=\frac{35}{96}=0.36458$

Part (c):

If annual income and age were independent, the probabilities in (a) and (b) would be equal. Since these probabilities are not equal, annual income and age category are not independent for adults in this sample.

Scoring

Part (a) is scored as either essentially correct (E) (may be minor arithmetic errors) or incorrect (I).

Part (b) is

Essentially correct (E) if the conditional probability is correctly calculated.

Partially correct (P) if the student reverses the conditioning, calculating

$P(income\ over\ 50,000 \mid age\ 31-45)=\frac{35}{89}=0.3933$

OR

calculates the correct probability for the wrong column, e.g., $\frac{32}{64}$

Incorrect (I) if the student calculates the joint probability: $\frac{35}{207}=0.169$

Part (c) is

Essentially correct (E) if the student

1. indicates that the two variables are not independent
   AND
2. the explanation is tied to the fact that the probabilities in parts (a) and (b) are not equal (the answer must be based on parts (a) and (b))

Question 2 (cont'd)

Partially correct (P) if the student indicates that the two variables are not independent, but the explanation is incorrect, or is not based on the answers to parts (a) and (b); i.e., performing new correct calculations instead of referring to those in parts (a) and (b). For example: determining the probability of the intersection and comparing to the two individual probabilities

$\left(\frac{35}{207} = 0.169, \text{ which does not equal } \frac{96}{207} \cdot \frac{89}{207} = (0.43)(0.46)\right)$, reversing conditions

$\left(\text{e.g., } \frac{96}{207} = 0.464, \text{ which does not equal } \frac{35}{89} = 0.393\right)$, or other conditional probability comparisons.

Incorrect (I) if the student fails to give a numerical justification to support the argument.
OR
Incorrect if the student does one of the following

- performs an incorrect additional calculation
- says the variables are independent based entirely on the context.
- performs a chi-square test ($\chi^2 = 5.38$, p-value = 0.496) since this addresses independence in the population instead of the sample
- only states "yes, independent" with no justification

NOTE: If either of the probabilities calculated in (a) or (b) are incorrect, part (c) should be scored as if those probabilities were correct. For example, if the student incorrectly calculated the same answer for parts (a) and (b), part (c) would be scored as correct if the student states that you can't tell if the two variables are independent because you would need to check all age-gender combinations.

4 Complete Response (EEE)

All three parts essentially correct

3 Substantial Response (EEP, EPE, EPP, IEE)

Part (a) essentially correct and parts (b) and (c) at least partially correct
OR
Part (a) incorrect and parts (b) and (c) essentially correct

2 Developing Response (EEI, EIE, EPI, EIP, IEP, IPE, IPP)

Part (a) essentially correct and one (but not both) of parts (b) and (c) correct
OR
Part (a) incorrect and both parts (b) and (c) at least partially correct

1 Minimal Response (EII, IPI, IIP, IEI, IIE)

Part (a) essentially correct and parts (b) and (c) incorrect
OR
Part (a) incorrect and one of parts (b) and (c) partially correct

#3

AP® STATISTICS
2002 SCORING GUIDELINES (Form B)

Question 2

Solution

Part (a):

P (everyone gets a seat) = P ($X \leq 38$) = .46 + .30 + .16 = .92

OR = 1 – (.05 + .02 + .01) = .92

Part (b):

Y = number of no shows

y	0	1	2	3	4	5
p(y)	.01	.02	.05	.16	.30	.46

E(Y) = 0(.01) + 1(.02) + 2(.05) + 3(.16) + 4(.30) + 5(.46) = 4.1

OR

E(X) = 36(.46) + 37(.30) + 38(.16) + 39(.05) + 40(.02) + 41(.01) = 36.9

E(Y) = 41 – E(X) = 4.1

Part (c):

P(X=36|not all seats are filled) = $P(X = 36 \mid X < 38) = \frac{P(X = 36)}{P(X < 38)} = \frac{.46}{.76} = .605$

Question 2 (cont'd.)

Scoring

Part (a) is

Essentially correct if computes P($X \leq 38$) (except for minor arithmetic errors)

Partially correct if computes P($X = 38$) = 0.16 or P($X > 38$) = .08 or P($X < 38$) = 0.76 or $P(X \geq 38) = 0.24$ or gives 0.92 but does not show any work.

Incorrect if no calculation or nonsensical calculation (e.g., 0.921, $\frac{38}{41} = 0.927$, $\frac{36.9}{38} = 0.971$) or only pure expected value calculations.

Part (b) is

Essentially correct if (except for minor arithmetic errors) correctly computes expected value for number of no shows and indicates fully where the 36.9 and 4.1 come from.

Rounding this value to 4 is considered a minus, though can be forgiven with "about" or "approximately."

Partially correct if correctly computes expected value of X = number of passengers who show up for the flight (instead of no shows) and shows work,

OR if incorrectly computes the first expected value but "subtracts,"

OR if correctly computes the first expected value but "subtracts" from the wrong number,

OR if does not show work for 36.9 but subtracts to get 4.1.

Incorrect if is not an expected value or does not use all six outcomes in the expected value.

Part (c) is

Essentially correct if correctly computes the conditional probability. Complete notation could be considered a "plus."

Partially correct if correctly computes $P(X = 36 \mid X < 41) = \frac{0.46}{0.99} = 0.465$ or $P(X = 36 \mid X \leq 38) = \frac{0.46}{0.92} = 0.5$ or incorrectly tries to solve the conditional probability (e.g., multiplies probabilities in numerator).

Incorrect if an unconditional probability is computed.

Question 2 (cont'd.)

4 Complete Response

Essentially correct on all three parts.

3 Substantial Response

Essentially correct on two parts and partially correct on the other part or incorrect on (a) and essentially correct on (b) and (c).

2 Developing Response

Essentially correct on two parts and incorrect on the other (except IEE – see above).
OR
Essentially correct on one part and partially correct on the other two parts.
OR
Partially correct on all three parts.
OR
Essentially correct on one part, partial on another, and incorrect on a third part.

1 Minimal Response

Essentially correct on one part and in correct on the other two parts.
OR
Partially correct on one or two parts and incorrect on the other.
PII should be graded holistically (needs something elsewhere for a 1).