Name: ____________________________________

STATISTICS

PART 4 PRACTICE EXAM 3

Time – 1 hour and 30 minutes

Number of multiple choice questions – 20

Number of free response questions – 3

1.

Two hundred students were classified by gender and hostility level (low, medium, high), as measured by an HLT-test. The results were the following:

	Hostility Level		
	Low	Medium	High
Male	35	40	5
Female	62	50	8

If the hostility level among students were independent of their gender, then how many female students would we expect to show the medium HLT score?

(A) 25
(B) 45
(C) 54
(D) 60
(E) 75

2.

In the jury pool available for this week, 30 percent of potential jurors are women. If a jury of 12 is to be selected at random, what is the expected number of men in the group?

(A) 12(0.30)

(B) 12(0.50)

(C) 12(0.70)

(D) 12(0.30)(0.70)

(E) $\sqrt{12(0.30)(0.70)}$

3.

A medicine is known to produce side effects in 1 in 5 patients taking it. Suppose a doctor prescribes the medicine to 4 unrelated patients. What is the probability that none of the patients will develop side effects?

(A) 0.8000
(B) 0.4096
(C) 0.2500
(D) 0.2000
(E) 0.0016

4.

An automobile service station performs only oil changes and tire replacements. Eighty percent of its customers request an oil change. Of those who request an oil change, only 20 percent request a tire replacement. What is the probability that the next customer will request both an oil change and a tire replacement?

(A) 0.16
(B) 0.20
(C) 0.25
(D) 0.80
(E) 0.85

5.

Which of the following is a discrete random variable?

(A) The number of times a student guesses the answers to questions on a certain test
(B) The amount of gasoline purchased by a customer
(C) The amount of mercury found in fish caught in the Gulf of Mexico
(D) The height of water-oak trees
(E) The time elapsed until the first field goal at home football games

6.

In which of the following situations is a binomial not an appropriate model to describe the outcome?

(A) The number of heads in three tosses of a coin
(B) The number of rainy days in a given week
(C) The number of girls in a family of 5 children
(D) The number of students present in a class of 22
(E) The number of defective computer monitors out of 7 purchased

7.

Which of the following statements about any two events A and B is true?

(A) $P(A \cup B)$ implies events A and B are independent.
(B) $P(A \cup B) = 1$ implies events A and B are mutually exclusive.
(C) $P(A \cap B) = 0$ implies events A and B are independent.
(D) $P(A \cap B) = 0$ implies events A and B are mutually exclusive.
(E) $P(A \cap B) = P(A) - P(B)$ implies A and B are equally likely events.

7.

Let X be a random variable that follows a t-distribution with a mean of 75 and a standard deviation of 8. Which of the following is (are) equivalent to $P(X > 85)$?

I. $P(X < 65)$
II. $P(X \geq 65)$
III. $1 - P(X < 65)$

a. I only
b. II only
c. III only
d. I and III only
e. I, II, and III

8.

Given $P(\text{A}) = 0.4$, $P(\text{B}) = 0.3$, $P(\text{B}|\text{A}) = 0.2$. What are $P(\text{A and B})$ and $P(\text{A or B})$?

a. $P(\text{A and B}) = 0.12$, $P(\text{A or B}) = 0.58$
b. $P(\text{A and B}) = 0.08$, $P(\text{A or B}) = 0.62$
c. $P(\text{A and B}) = 0.12$, $P(\text{A or B}) = 0.62$
d. $P(\text{A and B}) = 0.08$, $P(\text{A or B}) = 0.58$
e. $P(\text{A and B}) = 0.08$, $P(\text{A or B}) = 0.70$

Use this information for problems 10 and 11

At a local community college, 90% of students take English. 80% of those who don't take English take art courses, while only 50% of those who do take English take art.

10.

What is the probability that a student takes art?

a. 0.80
b. 0.53
c. 0.50
d. 1.3
e. 0.45

11.

What is the probability that a student who takes art doesn't take English?

a. 0.08
b. 0.10
c. 0.8
d. 0.85
e. 0.15

12.

Tom's career batting average is 0.265 with a standard deviation of 0.035. Larry's career batting average is 0.283 with a standard deviation of 0.029. The distribution of both averages is approximately normal. They play for different teams and there is reason to believe that their career averages are independent of each other. For any given year, what is the probability that Tom will have a higher batting average than Larry?

a. 0.389
b. 0.345
c. 0.589
d. 0.655
e. You cannot answer this question since the distribution for the difference between their averages cannot be determined from the data given.

13.

Which of the following statements is (are) correct?

I. The area under a probability density curve for a continuous random variable is 1.
II. A random variable is a numerical outcome of a random event.
III. The sum of the probabilities for a discrete random variable is 1.

a. II only
b. I and II
c. I and III
d. II and III
e. I, II, and III

14.

Given $P(\text{A}) = 0.60$, $P(\text{B}) = 0.30$, and $P(\text{A}|\text{B}) = 0.50$. Find $P(\text{A U B})$.

a. 0.90
b. 0.18
c. 0.40
d. 0.72
e. 0.75

15.

Let X be the number of points awarded for winning a game that has the following probability distribution:

X	0	2	3
P(X)	0.2	0.5	0.3

Let Y be the random variable whose sum is the number of points that results from two independent repetitions of the game. Which of the following is the probability distribution for Y?

a.

Y	0	2	3
P(Y)	0.2	0.5	0.3

b.

Y	0	4	6
P(Y)	0.2	0.5	0.3

c.

Y	0	2	3	4	6
P(Y)	0.2	0.25	0.15	0.25	0.15

d.

Y	0	2	3	4	5	6
P(Y)	0.04	0.2	0.12	0.25	0.3	0.09

e.

Y	0	2	3	4	5	6
P(Y)	0.04	0.10	0.06	0.25	0.15	0.09

16.

You play a game that involves rolling a die. You either win or lose $1 depending on what number comes up on the die. If the number is even, you lose $1, and if it is odd, you win $1. However, the die is weighted and has the following probability distribution for the various faces:

Face	1	2	3	4	5	6
Win (x)	+1	−1	+1	−1	+1	−1
$p(x)$	0.15	0.20	0.20	0.25	0.1	0.1

Given that you win rather than lose, what is the probability that you rolled a "5"?

a. 0.50
b. 0.10
c. 0.45
d. 0.22
e. 0.55

17.

It is the morning of the day that Willie and Baxter have planned their long-anticipated picnic. Willie reads, with some distress, that there is a 65% probability of rain in their area today. Which of the following best describes the most likely way that probability was arrived at?

a. It rains 65% of the time on this date each year.
b. Historically, in the United States, it has rained 65% of the time on days with similar meteorological conditions as today.
c. Historically, it rains 65% of the days during this month.
d. Historically, in this area, it has rained 65% of the time on days with similar meteorological conditions as today.
e. This is the result of a simulation conducted by the weather bureaus.

Use the following information for questions 18 and 19

Baxter is a 60% free-throw shooter who gets fouled during a game and gets to shoot what is called a "one-and-one" (that is, he gets to take a second shot—a bonus—if and only if he makes his first shot; each free throw, if made, is worth one point). Baxter can make 0 points (because he misses his first shot), 1 point (he makes the first shot, but misses the bonus), or 2 points (he makes his first shot and the bonus).

18.

Assuming that each shot is independent, how many points is Baxter *most likely* to make in a one-and-one situation?

a. 2
b. 1
c. 0
d. 0.96
e. None of these is correct.

19.

Assuming that each shot is independent, how many points will Baxter make *on average* in a one-and-one situation?

a. 2
b. 0.96
c. 0
d. 1
e. 0.36

20.

A fair die is to be rolled 8 times. What is the probability of getting at least one 4?

a. $\frac{1}{6}$

b. $\binom{8}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^7$

c. $1-\binom{8}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^7$

d. $\binom{8}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^6+\binom{8}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^5+\cdots+\binom{8}{8}\left(\frac{1}{6}\right)^8\left(\frac{5}{6}\right)^0$

e. $1-\binom{8}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^8$

FREE RESPONSE

Questions 1-3

Spend about 45 minutes on this part of the exam.

1.

Two antibiotics are available as treatment for a common ear infection in children.

- Antibiotic A is known to effectively cure the infection 60 percent of the time. Treatment with antibiotic A costs $50.
- Antibiotic B is known to effectively cure the infection 90 percent of the time. Treatment with antibiotic B costs $80.

The antibiotics work independently of one another. Both antibiotics can be safely administered to children. A health insurance company intends to recommend one of the following two plans of treatment for children with this ear infection.

- Plan I: Treat with antibiotic A first. If it is not effective, then treat with antibiotic B.
- Plan II: Treat with antibiotic B first. If it is not effective, then treat with antibiotic A.

(a) If a doctor treats a child with an ear infection using plan I, what is the probability that the child will be cured?

If a doctor treats a child with an ear infection using plan II, what is the probability that the child will be cured?

(b) Compute the expected cost per child when plan I is used for treatment.

Compute the expected cost per child when plan II is used for treatment.

(c) Based on the results in parts (a) and (b), which plan would you recommend?

Explain your recommendation.

2.

A simple random sample of adults living in a suburb of a large city was selected. The age and annual income of each adult in the sample were recorded. The resulting data are summarized in the table below.

	Annual Income			
Age Category	\$25,000-\$35,000	\$35,001-\$50,000	Over \$50,000	Total
21-30	8	15	27	50
31-45	22	32	35	89
46-60	12	14	27	53
Over 60	5	3	7	15
Total	47	64	96	207

(a) What is the probability that a person chosen at random from those in this sample will be in the 31-45 age category?

(b) What is the probability that a person chosen at random from those in this sample whose incomes are over \$50,000 will be in the 31-45 age category? Show your work.

(c) Based on your answers to parts (a) and (b), is annual income independent of age category for those in this sample? Explain.

3.

Airlines routinely overbook flights because they expect a certain number of no-shows. An airline runs a 5 P.M. commuter flight from Washington, D.C., to New York City on a plane that holds 38 passengers. Past experience has shown that if 41 tickets are sold for the flight, then the probability distribution for the number who actually show up for the flight is as shown in the table below.

Number who actually show up	36	37	38	39	40	41
Probability	0.46	0.30	0.16	0.05	0.02	0.01

Assume that 41 tickets are sold for each flight.

(a) There are 38 passenger seats on the flight. What is the probability that all passengers who show up for this flight will get a seat?

(b) What is the expected number of no-shows for this flight?

(c) Given that not all passenger seats are filled on a flight, what is the probability that only 36 passengers showed up for the flight?