Name: Key

STATISTICS

PART 4 PRACTICE EXAM 2

Time – 1 hour and 30 minutes

Number of multiple choice questions – 20

Number of free response questions - 3

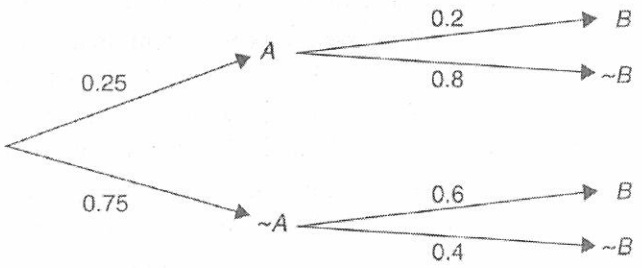
1.

What is the probability that on five rolls of a fair die you will roll three or more 1s?

(A) 99.7%
(B) 96.5%
(C) 40.2%
(D) 3.5%
(E) 0.3%

2.

Given the tree diagram shown, what is the probability that event B will happen given that event A has already occurred?



(A) 10%
(B) 20%
(C) 45%
(D) 50%
(E) 75%

3.

If $P(A) = 0.7$, $P(not\ B) = 0.4$, and $P(A \text{ and } B) = 0.5$, find $P(A \text{ or } B)$.

(A) 1.3
(B) 0.6
(C) 0.8
(D) 0.1
(E) 1.1

4.

There are 100 students enrolled in various AP* courses at Addison High School. There are 31 students enrolled in AP* Statistics, 52 students enrolled in AP* English, and 15 students enrolled in AP* French. Ten students study both AP* Statistics and AP* English, 5 students study both AP* Statistics and AP* French, 8 students study both AP* English and AP* French, and 3 students study all three AP* subjects. What is the probability that a student takes an AP* course other than these three?

(A) 9%
(B) 12%
(C) 22%
(D) 78%
(E) 93%

5.

Only 6 out of every 100 people have blood type O^+. What is the probability that in a random sample of individuals, the first person with type O^+ blood will be the eighth person tested?

(A) $C(8, 8)(0.06)^8(0.94)^0$
(B) $C(8, 1)(0.06)^1(0.94)^7$
(C) $C(1, 8)(0.06)^1(0.94)^7$
(D) $(0.06)^1(0.94)^7$
(E) $(0.06)^7(0.94)^1$

6.

Mr. DeVeaux teaches two sections of AP* Physics. He has 38 seniors in one section and 24 juniors in the other section. The overall mean for both sections on the midterm exam was 87. If the junior section had a mean of 92, what was the approximate mean for the senior section on the midterm exam?

(A) 82.6
(B) 83.8
(C) 89.5
(D) 87.0
(E) 90.4

7.

Statistics show that 7.3% of workers between the ages of 16 and 24 earn the minimum wage or less. What is the probability that if three young adults between the ages of 16 and 24 are polled, two or more will earn the minimum wage or less?

(A) 0.0004
(B) 0.0148
(C) 0.0152
(D) 0.0627
(E) 0.0677

8.

Of the registered voters in a community, 58% are female. A local politician running for office has the support of 48% of the registered women and 53% of the registered men. What percentage of the vote can the politician expect to get?

(A) 49.8%
(B) 50.1%
(C) 50.5%
(D) 58.58%
(E) Not enough information is given to determine the percentage of support for the politician.

9.

If $P(B) = 0.4$ and $P(A \cap B) = 0.21$, then find $P(A)$ if A and B are independent.

(A) 0.084
(B) 0.475
(C) 0.525
(D) 0.600
(E) Not possible

10.

Ruth plans to sell the jewelry she makes at an outdoor craft festival this coming Saturday. Based on her experience from past years, she can expect to make a profit of $400 if it is a sunny day, $275 if the weather is overcast, and $100 if it is raining. The weather forecaster (based on historical records) has estimated the chance of a sunny day for the day of the craft festival to be 0.65, the chance of an overcast day to be 0.15, and the chance of a rainy day to be 0.20. What is Ruth's expected profit from the sale of her jewelry?

(A) $400.00
(B) $321.25
(C) $275.00
(D) $258.33
(E) $100.00

11.

The probability that a car will skid on a bridge on a rainy day is 0.75. Today the weather station announced that there is a 20 percent chance of rain. What is the probability that it will rain today and that a car will skid on the bridge?

(A) 0.0300
(B) 0.0375
(C) 0.1500
(D) 0.3000
(E) 0.9500

12.

Suppose we have a random variable X with probability p. The probability of exactly 3 successes in 8 trials is given by

$$P(X = 3) = \binom{8}{3}(p)^3(0.45)^5.$$

What are the mean and standard deviation of X?

(A) mean = 4.4; standard deviation = 0.2475
(B) mean = 3.6; standard deviation = 0.2475
(C) mean = 4.4; standard deviation = 1.4071
(D) mean = 3.6; standard deviation = 1.4071
(E) There is not enough information to find the mean and standard deviation.

13.

For the given probability distribution, find the standard deviation of X.

X	1	3	5	7	9
$P(x)$	0.13	0.17	0.25	0.24	0.21

(A) 0.050
(B) 2.621
(C) 3.162
(D) 5.460
(E) 6.868

14.

A nursery guarantees that it will replace all the plants it sells that do not survive one year from the purchase date. From past experience the manager knows that 95 percent of plants sold survive more than a year. Suppose your school purchased 200 plants from this nursery to beautify the campus. How many plants do you expect will be replaced within a year?

(A) 5
(B) 10
(C) 20
(D) 95
(E) 190

15.

A publisher used standard boxes for shipping books. The mean weight of books packed per box is 25 pounds, with a standard deviation of 2 pounds. The mean weight of the boxes is 1 pound, with a standard deviation of 0.15 pounds. The mean weight of the packing material used per box is 2 pounds, with a standard deviation of 0.25 pounds. What is the standard deviation of the weights of the packed boxes?

(A) 28.000 pounds
(B) 5.290 pounds
(C) 4.085 pounds
(D) 2.400 pounds
(E) 2.021 pounds

16.

Which of the following is an outcome of a binomial experiment?

(A) Getting both spades on the first two draws from a standard deck of cards, when the first card is not replaced before the second card is drawn.
(B) Getting three spades out of the first seven draws from a standard deck of cards, when each card drawn is not replaced before the next card is drawn.
(C) Getting three spades out of the first seven draws from a standard deck of cards, when each card drawn is replaced before the next card is drawn.
(D) Getting three spades and four hearts out of the first seven draws from a standard deck of cards, when each card is not replaced before the next card is drawn.
(E) Getting three spades and four hearts out of the first seven draws from a standard deck of cards, when each card is replaced before the next card is drawn.

17.

A dentist has noticed that about 2 kids in every 7 that he sees professionally develop cavities before they turn 10 years old. Last week he examined the teeth of 5 unrelated children younger than 10. Let X be the number of children who develop cavities before turning 10. Which of the following gives the probability that at least one will develop a cavity before turning 10?

(A) $P(X = 2, 3, 4, 5, 6, 7)$
(B) $P(X = 2 \text{ out of } 7)$
(C) $P(X = 1)$
(D) $1 - P(X = 0)$
(E) $P(X = 0, 1)$

18.

Which of the following is true about any discrete probability distribution of a random variable X?

(A) The expected value of $X = np$.
(B) The sum of all possible values of X is equal to 1.
(C) The probabilities of all possible values of X must add up to 1.
(D) The probability distribution is bell-shaped and symmetric.
(E) Approximately 95 percent of the values of X fall within 2 standard deviations of the mean.

19.

The probability that Ted enrolls in an English class is $\frac{1}{3}$. If he does enroll in an English class, the probability that he enrolls in a mathematics class is $\frac{1}{5}$. What is the probability that he enrolls in both classes?

(A) $\frac{1}{15}$

(B) $\frac{2}{15}$

(C) $\frac{7}{15}$

(D) $\frac{3}{5}$

(E) $\frac{13}{15}$

20.

Two dice are rolled simultaneously. If both dice show 6, then the player wins \$20; otherwise the player loses the game. It costs \$2.00 to play the game. What is the expected gain or loss for the game?

(A) The player will gain about \$0.55.

(B) The player will gain about \$1.44.

(C) The player will lose about \$0.55.

(D) The player will lose about \$1.44.

(E) The player will lose about \$2.00.

Question 3

Intent of Question

The primary goals of this question are to assess a student's ability to: (1) recognize the random variable of interest, identify its probability distribution, and calculate a probability for a linear combination of a normal random variable and a constant; (2) use basic probability rules to find a different probability; and (3) use the sampling distribution of the sample mean to find a probability about the mean of three observations.

Solution

Part (a):

Since $M = D + E$ (a normal random variable plus a constant is a normal random variable), we know that M is normally distributed with a mean of 2 feet and a standard deviation of 1.5 feet. Thus,

$$P(M < 0) = P\left(Z < \frac{0-2}{1.5}\right) < P(Z < -1.33) = 0.0918, \text{ where } Z = \frac{M-\mu}{\sigma}.$$

Part (b):

$$\begin{aligned} P(\text{at least one measurement} < 0) &= 1 - P(\text{all three measurements} \geq 0) \\ &= 1 - (1 - 0.0918)^3 \\ &= 1 - (0.9082)^3 \\ &= 1 - 0.7491 \\ &= 0.2509 \end{aligned}$$

Part (c):

Let $\bar{X}$ denote the mean of three independent depth measurements taken at a point where the true depth is 2 feet. Since each measurement comes from a normal distribution, the distribution of $\bar{X}$ is normal with a mean of 2 feet and a standard deviation of $\frac{1.5}{\sqrt{3}} = 0.8660$ feet. Thus,

$$P(\bar{X} < 0) = P\left(Z < \frac{0-2}{\frac{1.5}{\sqrt{3}}}\right) < P(Z < -2.31) = 0.0104, \text{ where } Z = \frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}}.$$

Question 3 (continued)

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is essentially correct (E) if the student clearly does ALL three of the following:

- identifies the distribution as normal;
- specifies BOTH μ and σ; AND
- calculates the correct probability.

Part (a) is partially correct (P) if the student:

- calculates the correct probability but fails to identify the distribution as normal with BOTH μ and σ specified;
  OR
- correctly identifies the distribution as normal with BOTH μ and σ specified but fails to calculate the correct probability.

Part (a) is incorrect (I) if any of the following occur:

- the student indicates the probability is 0.5 because the random error is symmetric about zero;
  OR
- the student uses a mean of zero and a standard deviation of 1;
  OR
- the student conducts a hypothesis test.

Notes:

- The student may use the distribution of the error, E, to solve the problem. That is, finding the area below -2 for a normal distribution with mean 0 and standard deviation 1.5 should be scored essentially correct (E).
  Thus $P(E < -2) = P\left(Z < \frac{-2-0}{1.5}\right) < P(Z < -1.33) = 0.0918$, where $Z = \frac{E-\mu}{\sigma}$.
- If only the calculator command normalcdf $(-\infty, 0, 2, 1.5)$ is provided along with the probability 0.0912, then the response should be scored as partially correct (P).

Part (b) is essentially correct (E) if the student calculates the correct probability AND:

- correctly applies complement and probability rules using the value obtained in part (a);
  OR
- clearly identifies the distribution as binomial AND specifies BOTH n and p using the value obtained in part (a).

Part (b) is partially correct (P) if the student:

- clearly identifies the distribution as binomial AND specifies BOTH n and p, using the value obtained in part (a), but does not calculate the correct probability;
  OR

Question 3 (continued)

- clearly identifies the distribution as binomial AND specifies BOTH n and p using a value of p that is unrelated to the value obtained in part (a) and calculates the correct probability based on their value of p;
  OR
- calculates the correct probability using the value obtained in part (a) but fails to correctly identify the distribution as binomial with BOTH n and p specified;
  OR
- recognizes the solution as the sum of the product of the probabilities of successes and failures, using the answer from part (a), but omits only the binomial coefficients.

Part (b) is incorrect (I) if the student calculates $P(\text{at least one measurement} < 0) = 1 - p^3$, where p is the solution to part (a).

Notes:

- The solution using the binomial distribution with $p = 0.0918$ is:

$$\begin{aligned} P(\text{at least one measurement} < 0) &= P(B = 1) + P(B = 2) + P(B = 3) \\ &= \binom{3}{1} 0.0918^1 (1 - 0.0918)^2 + \\ &\quad \binom{3}{2} 0.0918^2 (1 - 0.0918)^1 + \binom{3}{3} 0.0918^3 \\ &= 0.2272 + 0.0230 + 0.0008 \\ &= 0.2510 \end{aligned}$$

- If only the calculator command $1 - \text{binomcdf}(3, 0.0918, 0)$ is provided along with the probability 0.2509, then the response should be scored as partially correct (P).

Part (c) is essentially correct (E) if the student clearly does ALL three of the following:

- identifies the distribution of the sample mean as normal;
- specifies BOTH $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$; AND
- calculates the correct probability.

Part (c) is partially correct (P) if the student:

- calculates the correct probability, but fails to identify the distribution of the sample mean as normal with BOTH $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$ specified;
  OR
- correctly identifies the distribution of the sample mean as normal with BOTH $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$ specified, but fails to calculate the correct probability.

Part (c) is incorrect (I) if any of the following occur:

- the student uses the same calculation as in part (a);
  OR

Question 3 (continued)

- the student uses an incorrect standard deviation (e.g., $\frac{1.5}{\sqrt{2}}$ or $\sqrt{3(1.5)}$;

  OR
- the student conducts a hypothesis test.

Notes:

- An alternate solution using the sum instead of the mean is:
  Let $\bar{X}$ denote the mean of three independent depth measurements taken at a point where the true depth is 2 feet. Since each measurement comes from a normal distribution, the distribution of the sum of the three measurements, $S = (X_1 + X_2 + X_3)$, is normal with a mean $\mu_S = 6$ feet and a standard deviation $\sigma_S = 2.598$ feet $\left(\sigma_S = 3\left(\frac{1.5}{\sqrt{3}}\right)\right.$, often calculated as $\left.\sqrt{(1.5)^2 + (1.5)^2 + (1.5)^2}\right)$.

  Thus $P(S < 0) = P\left(Z < \frac{0 - 6}{2.598}\right) = P(Z < -2.31) = 0.0104$, where $Z = \frac{S - \mu_S}{\sigma_S}$.

- If only the calculator command normalcdf $(-\infty, 0, 2, 0.866)$ is provided along with the probability 0.01046, then the response should be scored as partially correct (P).
- If the student does not consistently specify a correct μ and σ from the same distribution, i.e., for the mean or the sum, the response should be scored at most partially correct (P).

Question 3 (continued)

4 Complete Response

All three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct and one part partially correct

OR

One part essentially correct and no parts partially correct

OR

No parts essentially correct and two parts partially correct

#2

Question 2

Solution

Part (a):

The mean of C is $0 \times 0.4 + 1 \times 0.3 + 2 \times 0.2 + 3 \times 0.1 = 1$.
The standard deviation of C is $\sqrt{(0-1)^2 \times 0.4 + (1-1)^2 \times 0.3 + (2-1)^2 \times 0.2 + (3-1)^2 \times 0.1} = 1$.

Part (b):

Let $T = C + A$, where A is the total number of adult tickets purchased by a single customer, denote the total number of tickets purchased by a single customer.
The mean of T is $\mu_T = \mu_C + \mu_A = 1 + 2 = 3$.
The standard deviation of T is $\sigma_T = \sqrt{\sigma_C^2 + \sigma_A^2} = \sqrt{1^2 + 1.2^2} = \sqrt{2.44} = 1.562$.

Part (c):

Let $M = 15 \times C + 25 \times A$ denote the total amount of money spent per purchase.
The mean of M is $\mu_M = 15\mu_C + 25\mu_A = 15 \times 1 + 25 \times 2 = \$65$.
The standard deviation of M is $\sigma_M = \sqrt{15^2\sigma_C^2 + 25^2\sigma_A^2} = \sqrt{225 \times 1^2 + 625 \times 1.2^2} = \sqrt{1125} = \$33.54$.

Scoring

Each part is scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is essentially correct (E) if both the mean and the standard deviation of C are calculated correctly and the work is shown, with the exception of minor arithmetic errors.

Part (a) is partially correct (P) if either the mean or the standard deviation of C is calculated correctly and the work is shown.

Note: The variance and the standard deviation of C are both 1. If the variance is reported instead of the standard deviation, the response is scored as (P).

Part (a) is incorrect (I) if both the mean and the standard deviation of C are calculated incorrectly OR if no work is shown.

Notes:
1. Unsupported answers will be scored as incorrect.
2. If the student incorrectly calculates the mean and/or standard deviation in part (a) and then correctly uses those values in parts (b) and (c), there will be no second penalty.

Question 2 (continued)

3. Standard notation for means $(\mu_C, \mu_A, \mu_T, \text{and } \mu_M)$, variances $(\sigma_C^2, \sigma_A^2, \sigma_T^2, \text{and } \sigma_M^2)$, and standard deviations $(\sigma_C, \sigma_A, \sigma_T, \text{and } \sigma_M)$ are acceptable without definition. If nonstandard notation, such as $p_C, p_A, p_T, \text{and } p_M$, is defined correctly for this problem, then it will be scored as essentially correct. Nonstandard notation, without a definition, will be scored at most partially correct.

Part (b) is essentially correct (E) if both the mean and the standard deviation of T are calculated correctly and the work is shown, with the exception of minor arithmetic errors.

Part (b) is partially correct (P) if either the mean or the standard deviation of T is calculated correctly.

Part (b) is incorrect (I) if both the mean and the standard deviation of T are calculated incorrectly OR no work is shown.

Part (c) is scored as essentially correct (E) if both the mean and the standard deviation of M are calculated correctly and the work is shown, with the exception of minor arithmetic errors.

Part (c) is partially correct (P) if either the mean or the standard deviation of M is calculated correctly.

Part (c) is incorrect (I) if both the mean and the standard deviation of M are calculated incorrectly OR if no work is shown.

4 Complete Response (3E)

All three parts essentially correct

3 Substantial Response (2E 1P)

Two parts essentially correct and one part partially correct

2 Developing Response (2E 0P or 1E 2P)

Two parts essentially correct and zero parts partially correct
OR
One part essentially correct and two parts partially correct

1 Minimal Response (1E 1P or 1E 0P or 0E 2P)

One part essentially correct and either zero parts or one part partially correct
OR
Zero parts essentially correct and two parts partially correct

#3

Question 2

Solution

Part (a):

The expected number of telephone lines in use by the technical support center at noon is:

$$E(X) = 0 \times 0.35 + 1 \times 0.2 + 2 \times 0.15 + 3 \times 0.15 + 4 \times 0.1 + 5 \times 0.05$$
$$= 1.6$$

Part (b):

We would expect the average based on 1,000 days to be closer to 1.6 than the first average based on 20 days. Both averages have the same expected value (1.6), but the variability for sample averages based on 1,000 days is smaller than the variability for sample averages based on 20 days.

Part (c):

The median of X is 1.

x	$P(X \leq x)$	$P(X \geq x)$
0	0.35	1.0
1	0.55	0.65
2	0.70	0.45
3	0.85	0.30
4	0.95	0.15
5	1.0	0.05

OR

The median of X is 1 because $P(X \leq 1) = 0.55 \geq 0.50$ and $P(X \geq 1) = 0.65 \geq 0.50$.

Part (d):

The probability histogram is clearly skewed to the right (or toward the larger values) so the mean (1.6) is larger than the median (1), as is typical for a right-skewed distribution.

Question 2 (continued)

Scoring

Parts (a) and (c) are combined as one computational part. Each part is scored as essentially correct (E), partially correct (P), or incorrect (I).

Collectively parts (a) and (c) are essentially correct (E) if both parts are calculated correctly, with the exception of minor arithmetic errors.

Collectively parts (a) and (c) are partially correct (P) if one of the two parts is calculated correctly, with the exception of minor arithmetic errors.

Collectively parts (a) and (c) are incorrect (I) if both parts are calculated incorrectly.

Note: Unsupported answers in parts (a) and (c) are scored as incorrect.

Part (b) is essentially correct (E) if the student:

1. States the new estimate based on 1,000 days should be closer to the expected value of 1.6; OR the new estimate will increase, or decrease if the answer in part (a) is less than 1.25.

AND

2. Provides justification by stating the variability for sample averages based on 1,000 days will be smaller than the variability for sample averages based on 20 days; OR as the sample size increases the sample average approaches the expected value of X.

Part (b) is partially correct (P) if the student provides one of the two items above.

Part (d) is essentially correct (E) if the student states that since the distribution is skewed to the right, the mean is greater than the median; OR since the mean is greater than the median, the distribution is skewed to the right.

Note: There must be evidence that the student looked at the given distribution.

Part (d) is partially correct (P) if the student:

- States that since the mean is greater than the median, the distribution is skewed to the right (with no evidence that the student looked at the given distribution); OR
- Compares the two measures of center by referring to the inappropriate or incomplete shape of the distribution (e.g., "skewed to the left" or "skewed"); OR
- Makes a correct statement about the measures of center and the shape without connecting the two.

Question 2 (continued)

Part (d) is incorrect (I) if the student:

- Compares the two measures of center without mentioning the shape of the distribution; OR
- Correctly describes the shape without correct conclusions about the relative location of the mean and median; OR
- Makes multiple "generic" statements about the relationship of mean, median, and shape with no reference to the given distribution.

4 Complete Response (3E)

All three parts essentially correct

3 Substantial Response (2E 1P)

Two parts essentially correct and one part partially correct

2 Developing Response (2E 0P or 1E 2P)

Two parts essentially correct and zero parts partially correct
OR
One part essentially correct and two parts partially correct

1 Minimal Response (1E 1P or 1E 0P or 0E 2P)

One part essentially correct and either zero parts or one part partially correct
OR
Zero parts essentially correct and two parts partially correct