

Name: _____

STATISTICS

PART 4 PRACTICE EXAM 1

Time – 1 hour and 30 minutes

Number of multiple choice questions – 20

Number of free response questions - 3

1.

Given two events, A and B , if $P(A) = 0.43$, $P(B) = 0.26$, and $P(A \cup B) = 0.68$, then the two events are

- (A) mutually exclusive but not independent.
- (B) independent but not mutually exclusive.
- (C) mutually exclusive and independent.
- (D) neither mutually exclusive nor independent.
- (E) Not enough information is given to determine whether A and B are mutually exclusive or independent.

2.

The number of T-shirts a school store sells monthly has the following probability distribution:

# of T-shirts, X	0	1	2	3	4	5	6	7	8	9	10
$P(X)$	0.02	0.15	0.18	0.21	0.14	0.08	0.08	0.04	0.03	0.02	0.05

If each T-shirt sells for \$10 but costs the store \$4 to purchase, what is the expected monthly T-shirt *profit*?

- (A) \$ 3.78
- (B) \$15.12
- (C) \$22.68
- (D) \$30.00
- (E) \$37.80

3.

A young woman works two jobs and receives tips for both jobs. As a hairdresser, her distribution of weekly tips has mean \$65 and standard deviation \$5.75. As a waitress, her distribution of weekly tips has mean \$154 and standard deviation \$8.02. What are the mean and standard deviation of her combined weekly tips? (Assume independence for the two jobs.)

- (A) mean \$167.16; standard deviation \$9.87
- (B) mean \$167.16; standard deviation \$13.77
- (C) mean \$219.00; standard deviation \$2.27
- (D) mean \$219.00; standard deviation \$9.87
- (E) mean \$219.00; standard deviation \$13.77

4.

Which of the following is not a condition for a geometric setting?

- (A) There are only two possible outcomes for each trial.
- (B) The probability of success is the same for each trial.
- (C) The trials are independent.
- (D) There are a fixed number of observations.
- (E) The variable of interest is the number of trials required to reach the first success.

5.

In a game of chance, three fair coins are tossed simultaneously. If all three coins show heads, then the player wins \$15. If all three coins show tails, then the player wins \$10. If it costs \$5 to play the game, what is the player's expected net gain or loss at the end of two games?

- (A) The player can expect to gain \$15 after two games.
- (B) The player can expect to gain \$1.88 after two games.
- (C) The player can expect to gain \$3.75 after two games.
- (D) The player can expect to lose \$1.88 after two games.
- (E) The player can expect to lose \$3.75 after two games.

6.

Senior citizens make up about 12.4% of the American population. If a random sample of 200 Americans is selected, what is the probability that more than 180 of them are *not* senior citizens?

(A) $\binom{200}{180}(0.124)^{180}(0.876)^{20}$

(B) $\binom{200}{180}(0.876)^{180}(0.124)^{20}$

(C) $P\left(z > \frac{180 - 175.2}{\frac{0.124}{\sqrt{200}}}\right)$

(D) $P\left(z > \frac{0.9 - 0.124}{\sqrt{\frac{(0.124)(0.876)}{200}}}\right)$

(E) $P\left(z > \frac{0.9 - 0.876}{\sqrt{\frac{(0.124)(0.876)}{200}}}\right)$

7.

As a promotional gimmick, a cereal manufacturer packages boxes of cereal with CD-ROMs of popular games. There are five different games distributed equally among the boxes, but the purchasers do not know which game they are receiving when they purchase the cereal. A child would like to receive one game in particular. What is the probability that the child opens three boxes of cereal before receiving the desired game?

(A) $\binom{5}{3}(0.2)^3(0.8)^2$

(B) $\binom{5}{3}(0.2)^2(0.8)^3$

(C) $\binom{5}{1}(0.6)(0.4)^4$

(D) $(0.8)^2(0.2)$

(E) $(0.2)^2(0.8)$

8.

Suppose the probability of encountering an American who practices a particular religion is 0.014. What are the mean and standard deviation for the *number* of Americans in a random sample of 500 who practice this religion?

(A) mean 0.014; standard deviation 0.0006

(B) mean 0.014; standard deviation 0.0053

(C) mean 7; standard deviation 0.0006

(D) mean 7; standard deviation 0.0053

(E) mean 7; standard deviation 2.627

9.

An airline has an on-time probability of 82.4%. What is the probability that, if you travel on this airline, no more than 3 of your next 10 flights will *not* be on time? (Assume that flights are independent.)

(A) $\binom{10}{3}(0.176)^3(0.824)^7$

(B) $\binom{10}{3}(0.824)^3(0.176)^7$

(C) $\binom{10}{0}(0.176)^0(0.824)^{10} + \binom{10}{1}(0.176)^1(0.824)^9 + \binom{10}{2}(0.176)^2(0.824)^8$

(D) $\binom{10}{0}(0.824)^0(0.176)^{10} + \binom{10}{1}(0.824)^1(0.176)^9 + \binom{10}{2}(0.824)^2(0.176)^8$

(E) $\binom{10}{0}(0.176)^0(0.824)^{10} + \binom{10}{1}(0.176)^1(0.824)^9 + \binom{10}{2}(0.176)^2(0.824)^8 + \binom{10}{3}(0.176)^3(0.824)^7$

10.

Owners of a day-care chain wish to determine the proportion of families in need of day care for the town of Bockville. Bockville is estimated to have 1000 families. The owners of the day-care chain randomly sample 50 families and find that 60% of them have a need for day-care services. Which of the following is a condition necessary for constructing a confidence interval for a **proportion** that has *not* been met?

- (A) The data constitute a representative random sample from the population of interest.
- (B) The sample size is less than 10% of the population size.
- (C) The counts of those who need day care and those who don't need day care are 10 or more.
- (D) The distribution of sample values is approximately normally distributed.
- (E) All conditions necessary for constructing a confidence interval for the proportion seem to be met.

11.

The cause of death and the age of the deceased are recorded for 454 patients from a hospital.

	15–24	25–34	35–44	45–54	55–64	Total
Accident	14	12	15	12	7	60
Homicide	5	4	3	0	0	12
Heart disease	1	3	14	34	63	115
HIV	0	3	6	4	0	13
Cancer	2	4	17	47	89	159
Other	3	7	16	26	43	95
Total	25	33	71	123	202	454

Use these values to estimate the probability that a person at this hospital died as a result of an accident if it is known the person was between the ages of 45 and 54.

- (A) 0.0264
- (B) 0.0976
- (C) 0.1322
- (D) 0.2000
- (E) 0.4878

12.

The following table shows the preferred exercise for a random sample of 223 men of various ages.

Physical Activity/Age	18–31	32–45	46–59	60–73	Over 74
Jogging	23	14	9	1	0
Cycling	19	19	14	11	8
Swimming	10	8	5	3	1
Weight Lifting	34	21	12	6	5

If the type of exercise is independent of age, how many men over the age of 74 would we expect to prefer cycling?

- (A) 3
- (B) 4
- (C) 8
- (D) 11
- (E) 14

13.

A baseball recruiter visits a high school where a player has a batting average of 0.450. (This means that he gets a hit in 45% of his at-bats.) What is the probability that the recruiter won't see the player get a hit until his third at-bat?

- (A) $(0.450)^2(0.550)$
- (B) $(0.550)^2(0.450)$
- (C) $\binom{3}{1}(0.450)(0.550)^2$
- (D) $\binom{3}{1}(0.550)(0.450)^2$
- (E) $\binom{3}{2}(0.450)(0.550)^2$

14.

If two events, A and B , are mutually exclusive, then the probability that both A and B occur simultaneously is

- (A) 0.
- (B) 1.
- (C) $P(A) + P(B)$.
- (D) $P(A) + P(B) - P(A \cap B)$.
- (E) $P(A)P(B)$.

15.

Pearson High School students have cumulative grade point averages as shown in the table.

Class \ GPA	≥ 4.0	3.0–4.0	2.0–3.0	1.0–2.0	< 1.0	Total
Sophomores	43	121	114	22	10	310
Juniors	26	102	84	16	5	233
Seniors	15	87	100	10	7	219
Total	84	310	298	48	22	762

Which of the following statements is *not* true?

- (A) About 39% of sophomores have *at least* a 3.0 GPA.
- (B) Sophomores represent 39% of GPAs from 3.0 to 4.0.
- (C) Seniors represent about 29% of the reported GPAs at Pearson High School.
- (D) Only about 3% of seniors have GPAs *less than* 1.0.
- (E) About 11% of the reported GPAs are juniors with GPAs from 2.0 to 3.0.

16.

Given the information below, which of the statements is true?

X	2	4	6	8	10
$P(X)$	0.3	0.2	0	0.4	0.1

- (A) The expected value of the random variable is 6.
- (B) The expected value of the random variable is 0.
- (C) The variance of the random variable is 1.
- (D) The expected value of the random variable is 11.6.
- (E) The variance of the random variable is 8.64.

17.

Two random variables, X and Y , are independent. X has expected value 2.5 and standard deviation 0.3, while Y has expected value 4.7 and standard deviation 0.4. Which of the following is true?

- (A) The mean of $X + Y$ is 6.2.
- (B) The standard deviation of $X + Y$ is 0.7.
- (C) The variance of $X + Y$ is 0.7.
- (D) The mean of $X + Y$ is 11.75.
- (E) The standard deviation of $X + Y$ is 0.5.

18.

Which of the following is true?

- (A) The value of a random variable must always be positive.
- (B) The expected value of a random variable must always be positive.
- (C) The variance of a random variable must always be positive.
- (D) The expected value of a random variable must be nonzero.
- (E) The variance of a random variable must be nonnegative (0 or positive).

19.

Home pregnancy test kits have grown in popularity. Research shows that only 30% of those using a particular kit are actually pregnant. When a pregnant woman uses this kit, it correctly indicates pregnancy 96% of the time. A woman who is not pregnant gets a correct indication 90% of the time. What is the probability that a woman is pregnant given that this test gives a positive result?

- (A) About 96%
- (B) About 86%
- (C) About 80%
- (D) About 36%
- (E) About 21%

20.

Two friends, Tom and Janice, have cars in desperate need of repair. On any given day, the probability that Tom's car will break down is 0.5, the probability that Janice's car will break down is 0.5, and the probability that both of their cars will break down is 0.3. What is the probability that Tom or Janice's car will break down?

- (A) 1.3
- (B) 1.0
- (C) 0.7
- (D) 0.4
- (E) 0.2

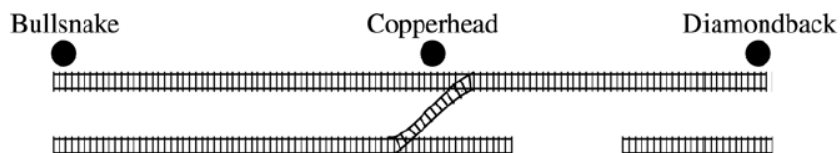
FREE RESPONSE

Questions 1-3

Spend about 45 minutes on this part of the exam.

1.

Flooding has washed out one of the tracks of the Snake Gulch Railroad. The railroad has two parallel tracks from Bullsnake to Copperhead, but only one usable track from Copperhead to Diamondback, as shown in the figure below. Having only one usable track disrupts the usual schedule. Until it is repaired, the washed-out track will remain unusable. If the train leaving Bullsnake arrives at Copperhead first, it has to wait until the train leaving Diamondback arrives at Copperhead.



Every day at noon a train leaves Bullsnake heading for Diamondback and another leaves Diamondback heading for Bullsnake.

Assume that the length of time, X , it takes the train leaving Bullsake to get to Copperhead is normally distributed with a mean of 170 minutes and a standard deviation of 20 minutes.

Assume that the length of time, Y , it takes the train leaving Diamondback to get to Copperhead is normally distributed with a mean of 200 minutes and a standard deviation of 10 minutes.

These two travel times are independent.

- (a) What is the distribution of $Y - X$?
(What is the mean and standard deviation)
- (b) Over the long run, what proportion of the days will the train from Bullsake have to wait at Copperhead for the train from Diamondback to arrive?

- (c) How long should the Snake Gulch Railroad delay the departure of the train from Bullsake so that the probability that it has to wait is only 0.01 ?

2.

A local arcade is hosting a tournament in which contestants play an arcade game with possible scores ranging from 0 to 20. The arcade has set up multiple game tables so that all contestants can play the game at the same time; thus contestant scores are independent. Each contestant's score will be recorded as he or she finishes, and the contestant with the highest score is the winner.

After practicing the game many times, Josephine, one of the contestants, has established the probability distribution of her scores, shown in the table below.

Josephine's Distribution				
Score	16	17	18	19
Probability	0.10	0.30	0.40	0.20

Crystal, another contestant, has also practiced many times. The probability distribution for her scores is shown in the table below.

Crystal's Distribution			
Score	17	18	19
Probability	0.45	0.40	0.15

(a) Calculate the expected score for each player.

(b) Suppose that Josephine scores 16 and Crystal scores 17. The difference (Josephine minus Crystal) of their scores is -1 . List all combinations of possible scores for Josephine and Crystal that will produce a difference (Josephine minus Crystal) of -1 , and calculate the probability for each combination.

- (c) Find the probability that the difference (Josephine minus Crystal) in their scores is -1 .

- (d) The table below lists all the possible differences in the scores between Josephine and Crystal and some associated probabilities.

Distribution (Josephine minus Crystal)						
Difference	-3	-2	-1	0	1	2
Probability	0.015			0.325	0.260	0.090

Complete the table and calculate the probability that Crystal's score will be higher than Josephine's score.

3.

Golf balls must meet a set of five standards in order to be used in professional tournaments. One of these standards is distance traveled. When a ball is hit by a mechanical device, Iron Byron, with a 10-degree angle of launch, a backspin of 42 revolutions per second, and a ball velocity of 235 feet per second, the distance the ball travels may not exceed 291.2 yards. Manufacturers want to develop balls that will travel as close to the 291.2 yards as possible without exceeding that distance. A particular manufacturer has determined that the distances traveled for the balls it produces are normally distributed with a standard deviation of 2.8 yards. This manufacturer has a new process that allows it to set the mean distance the ball will travel.

- If the manufacturer sets the mean distance traveled to be equal to 288 yards, what is the probability that a ball that is randomly selected for testing will travel too far?
- Assume the mean distance traveled is 288 yards and that five balls are independently tested. What is the probability that at least one of the five balls will exceed the maximum distance of 291.2 yards?
- If the manufacturer wants to be 99 percent certain that a randomly selected ball will not exceed the maximum distance of 291.2 yards, what is the largest mean that can be used in the manufacturing process?