Practice FRQ #1

AP® STATISTICS
2011 SCORING GUIDELINES

Question 2

Intent of Question

The primary goals of this question were to assess students' ability to (1) determine a conditional probability from a table of data; (2) use a table of data to determine whether or not two events are independent; (3) demonstrate an understanding of the concept of independence by constructing a graph that displays independence between two variables.

Solution

Part (a):

Of the 200 male registered voters in Franklin Township, 48 are registered for Party Y. Therefore the conditional probability that a randomly selected voter is registered for Party Y, given that the voter is a male, is $\frac{48}{200} = 0.24$.

Part (b):

No, the events "is a male" and "is registered for Party Y" are not independent. One justification of this conclusion is to note that the conditional probability of the event "is registered for Party Y" given the event "is a male" — which was computed in part (a) — is not equal to the probability of the event "is registered for Party Y," as shown below.

$$P(\text{is registered for Party Y} \mid \text{is a male}) = 0.24$$

$$P(\text{is registered for Party Y}) = \frac{168}{500} = 0.336$$

Because $0.24 \neq 0.336$, the two events are not independent.

Part (c):

The marginal proportions of voters registered for each of the three political parties (without regard to gender) are given below.

$$\text{Party W: } \frac{88}{500} = 0.176$$

$$\text{Party X: } \frac{244}{500} = 0.488$$

$$\text{Party Y: } \frac{168}{500} = 0.336$$

Because party registration is independent of gender in Lawrence Township, the proportions of males and females registered for each party must be identical to each other and also identical to the marginal proportion of voters registered for that party. Using the order Party W, Party X, and Party Y, the graph for Lawrence Township is displayed below.

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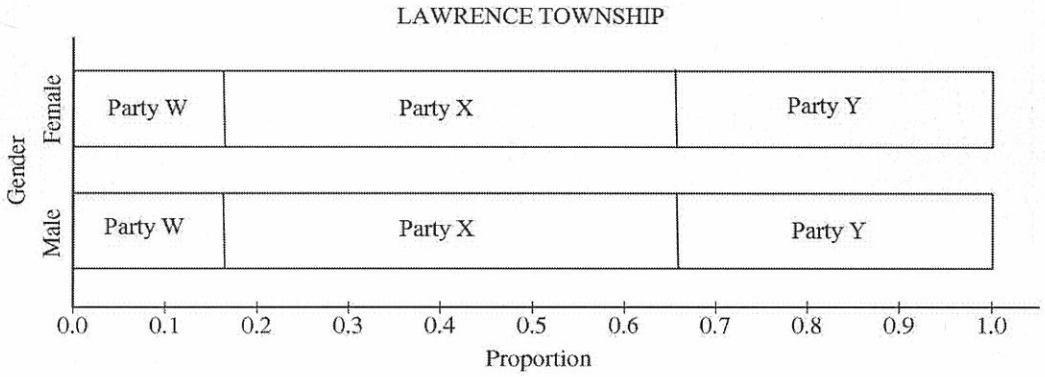
Question 2 (continued)



Scoring

Parts (a), (b) and (c) are scored as essentially correct (E), partially correct (P) or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response has the correct conditional probability *AND* shows the work.

Partially correct (P) if the response has the correct reverse conditional probability (of being a male given that he is registered for Party Y),
OR
if the response has the correct conditional probability *BUT* does not show work.

Incorrect (I) if the response fails to meet the criteria for E or P.

Part (b) is scored as follows:

Essentially correct (E) if the response identifies two values whose inequality implies a lack of independence between the events *AND* includes the following three components:
1. Correct computations of the two values.
2. An explicit statement of whether the two values are equal or unequal.
3. An appropriate conclusion about the independence of the events.

Partially correct (P) if the response identifies two values whose inequality implies a lack of independence between the events but includes only two of the three components listed above.

Incorrect (I) if the response fails to meet the criteria for E or P.

Part (c) is scored as follows:

Essentially correct (E) if the response shows the same conditional distribution of party registration for both males and females *AND* includes the following two components:
1. Correct proportions for each party.
2. Correct labels (Party W, Party X, Party Y).

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Question 2 (continued)

Partially correct (P) if the response shows the same conditional distribution of party registration for both males and females *AND* includes only one of the two components listed above.

Incorrect (I) if the response fails to meet the criteria for E or P.

Note: For all three parts, an incorrect statement that indicate a serious misunderstanding of statistical concepts, even if unrelated to the rest of the response, lowers the score one level (that is, from E to P, or from P to I). An example of this is a response that indicates confusion between independent events and disjoint events.

4 **Complete Response**

All three parts essentially correct

3 **Substantial Response**

Two parts essentially correct and one part partially correct

2 **Developing Response**

Two parts essentially correct and one part incorrect

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 **Minimal Response**

One part essentially correct and two parts incorrect

OR

Two parts partially correct and one part incorrect

Practice FRQ #2

AP® STATISTICS
2009 SCORING GUIDELINES

Question 2

Intent of Question

The primary goals of this question were to assess a student's ability to (1) calculate a percentile value from a normal probability distribution; (2) recognize a binomial scenario and calculate an appropriate probability; and (3) use the sampling distribution of the sample mean to find a probability for the mean of five observations.

Solution

Part (a):

Let X denote the stopping distance of a car with new tread tires where X is normally distributed with a mean of 125 feet and a standard deviation of 6.5 feet. The z-score corresponding to a cumulative probability of 70 percent is $z = 0.52$. Thus, the 70th percentile value can be computed as:

$x = \mu_X + z\sigma_X = 125 + 0.52(6.5) = 128.4$ feet.

Part (b):

From part (a), it was found that a stopping distance of 128.4 feet has a cumulative probability of 0.70. Thus the probability of a stopping distance greater than 128.4 is $1 - 0.70 = 0.30$.

Let Y denote the number of cars with the new tread pattern out of five cars that stop in a distance greater than 128.4 feet. Y is a binomial random variable with $n = 5$ and $p = 0.30$.

$$P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - \left[\binom{5}{0}(0.3)^0(0.7)^5 + \binom{5}{1}(0.3)^1(0.7)^4 \right]$$

$$= 1 - 0.5282 = 0.4718.$$

Part (c):

Let $\bar{X}$ denote the mean of the stopping distances of five randomly selected cars. All tires have the new tread pattern. Because the stopping distance for each of the five cars has a normal distribution, the distribution of $\bar{X}$ is normal with a mean of 125 feet and a standard deviation of $\frac{6.5}{\sqrt{5}} = 2.91$ feet. Thus,

$$P(\bar{X} > 130) = P\left(Z > \frac{130 - 125}{6.5/\sqrt{5}} \right) \approx P(Z > 1.72) = 0.0427.$$

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

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2009 SCORING GUIDELINES

Question 2 (continued)

Part (a) is scored as follows:

Essentially correct (E) if the student clearly indicates which distribution is being used, along with the parameters (μ_X and σ_X), and correctly calculates the percentile value with appropriate justification (except for minor arithmetic or transcription errors). There are three components: distribution, parameters, and calculation of distance.

Notes:

- The standard notation $N(125, 6.5)$ defines distribution and parameters. Also, the z-score formula setup implies distribution and parameters. This applies only in part (a), because approximate normality is given in the stem of the problem.
- If the calculator command invNorm(0.70, 125, 6.5) is provided along with 128.4 feet *AND* an appropriately labeled sketch of a normal distribution is supplied, then the response should be scored as essentially correct (E). An appropriately labeled sketch must include correct labels for center and spread.
- If the calculator command invNorm(0.70, $\mu = 125$, $\sigma = 6.5$) is provided along with 128.4 feet, then the response should be scored as essentially correct (E).

Partially correct (P) if the student correctly supplies only two out of the three components.

Note: If the calculator command invNorm(0.70, 125, 6.5) is provided along with 128.4 feet, then the response should be scored as partially correct (P).

Incorrect (I) if the student correctly supplies at most one of the components.

Part (b) is scored as follows:

Essentially correct (E) if the student recognizes this probability as an application of the binomial distribution and sets up the problem correctly by first finding the probability for p, the probability of a success, and then using this p to find the correct binomial probability. There are three components: distribution, parameters, and calculation.

Note: If the calculator command 1-binomcdf(5, 0.3, 1) is provided along with 0.4718 and an identification of the distribution and its parameters—e.g., by the standard notation $B(5,0.3)$ or Bin(5, 0.3)—then the response should be scored as essentially correct (E).

Partially correct (P) if the student correctly supplies only two out of the three components.

Notes:

- As long as the student identifies the distribution and parameters—e.g., by the standard notation $B(5, 0.3)$ or Bin(5, 0.3)—the binomial formula does not need to be set up to receive full credit. However, the binomial formula setup can suffice for identifying two of the three components: distribution and parameters.
- If the calculator command 1-binomcdf(5, 0.3, 1) is provided along with 0.4718, then the response should be scored as partially correct (P).

Incorrect (I) if the student correctly supplies at most one of the components.

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Question 2 (continued)

Part (c) is scored as follows:

Essentially correct (E) if the student recognizes that the distribution of the sample mean will be approximately normal with the appropriate mean and standard deviation and calculates the probability correctly. There are three components: sampling distribution, parameters, and calculation.

Notes:
- The z-score formula setup suffices only for parameters in part (c).
- If the calculator command Normalcdf(130, ∞, 125, 2.91) *AND* an appropriately labeled sketch of a normal distribution are provided along with the value obtained using the calculator, 0.0428, then the response should be scored as essentially correct (E). An appropriately labeled sketch must include correct labels for center and spread.
- If the calculator command Normalcdf(130, ∞, $\mu = 125$, $\sigma = 2.91$) is provided along with 0.0428, then the response should be scored as essentially correct (E).

Partially correct (P) if the student correctly supplies only two out of the three components.

Note: If the calculator command Normalcdf(130, ∞, 125, 2.91) is provided along with 0.0428, then the response should be scored as partially correct (P).

Incorrect (I) if the student correctly supplies at most one of the components.

Notes:
- The calculator solution is 0.0428. If this is the only information provided, the response is scored as incorrect (I).
- If a t distribution is used, then the response should be scored as incorrect (I).

A student should be penalized only once for using calculator syntax—that is, look at parts (a), (b), and (c) together.

4 **Complete Response**

All three parts essentially correct

3 **Substantial Response**

Two parts essentially correct and one part partially correct

2 **Developing Response**

Two parts essentially correct and no part partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

Practice FRQ #3

AP® STATISTICS
2010 SCORING GUIDELINES (Form B)

Question 3

Intent of Question

The primary goals of this question were to assess students' ability to (1) recognize binomial distribution scenarios and calculate relevant binomial probabilities; (2) calculate expected values based on the binomial distribution and properties of expectation.

Solution

Part (a):

Let X denote the number of correct guesses, assuming that a student guesses randomly among the five options on all 25 questions. Then X has a binomial probability distribution with $n = 25$ and $p = \frac{1}{5} = 0.20$.

Part (b):

Let Y denote the number of correct responses on the seven questions for which the student guesses randomly from among the five options. Then Y has a binomial probability distribution with $n = 7$ and $p = 0.20$. Then the expected value of Y, $\mathrm{E}(Y) = np = 7(0.20) = 1.4$ correct responses.

Next, using the scoring formula provided,
$\text{Score} = (18 + Y) - 0.25(7 - Y) + 0(0) = 16.25 + 1.25Y$.

The expected exam score is therefore:
$\mathrm{E}(\text{Score}) = \mathrm{E}(16.25 + 1.25Y) = 16.25 + 1.25\mathrm{E}(Y) = 16.25 + 1.25(1.4) = 16.25 + 1.75 = 18$ correct responses.

Part (c):

Let Y be defined as in part (b). The student passes when Score ≥ 20, which means that $16.25 + 1.25Y \geq 20$, which means that $Y \geq \frac{20 - 16.25}{1.25} = 3$. In other words, in order to pass, the student must get three or more correct from the seven questions on which the student guesses.

Y has a binomial probability distribution with $n = 7$ and $p = 0.20$, so

$$P(Y \geq 3) = 1 - P(Y \leq 2) = 1 - \left[\binom{7}{0}(.2)^0(.8)^7 + \binom{7}{1}(.2)^1(.8)^6 + \binom{7}{2}(.2)^2(.8)^5\right] = 1 - 0.852 = 0.148.$$

Scoring

Parts (a), (b) and (c) are each scored as essentially correct (E), partially correct (P) or incorrect (I).

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2010 SCORING GUIDELINES (Form B)

Question 3 (continued)

Part (a) is scored as follows:

Essentially correct (E) if the response identifies the correct type of probability distribution (binomial) *AND* identifies the two parameter values, $n = 25$ and $p = \frac{1}{5} = 0.20$, correctly. There are two components: naming the probability distribution and identifying the two parameter values.

Partially correct (P) if the response correctly identifies only one of the two components (either the name of the probability distribution or the parameter values).

Incorrect (I) if the response neither correctly names the distribution nor identifies both parameter values correctly.

Note: Notation such as $B(25,0.2)$ will be scored as essentially correct (E) for this part.

Part (b) is scored as follows:

Essentially correct (E) if the number of trials, n, and the binomial probability of a success p for Y (the number of correct guesses) are used to find the expected value of Y, $\mathrm{E}(Y)$, and if the correct expected exam score is calculated using $\mathrm{E}(Y)$. There are two components: calculating $\mathrm{E}(Y)$ and calculating the expected exam score.

Partially correct (P) if only one of the two components is correct.

Incorrect (I) if neither component is correct.

Part (c) is scored as follows:

Essentially correct (E) if the response specifies that three or more correct guesses are needed and the binomial probability is calculated correctly. There are two components: correctly identifying the required probability and correct calculation of the probability.

Partially correct (P) if only one of the two components is correct.

Incorrect (I) if neither component is correct.

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2010 SCORING GUIDELINES (Form B)

Question 3 (continued)

4 **Complete Response**

All three parts essentially correct

3 **Substantial Response**

Two parts essentially correct and one part partially correct

2 **Developing Response**

Two parts essentially correct and one part incorrect

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 **Minimal Response**

One part essentially correct and two parts incorrect

OR

Two parts partially correct and one part incorrect