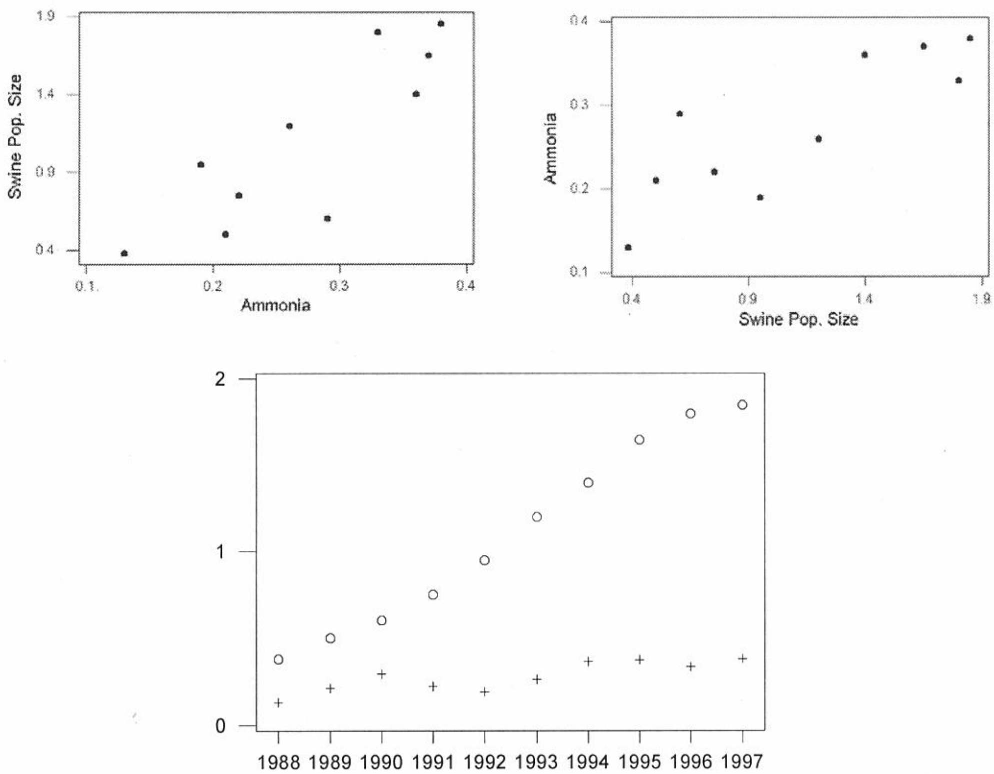
FR Practice 1

Question 1

Solution

Part (a):



Part (b):

There is a strong, positive, linear relationship between swine population size and atmospheric ammonia.

Part (c):

Both the value of the correlation coefficient and the pattern in the scatterplot indicate that there is a positive linear relationship between the size of the swine population and atmospheric ammonia.

Part (d):

r^2 = .72 or 72% or 72

Note: Just writing r^2 is not sufficient.

Question 1 (cont'd.)

Scoring

Part (a) is considered

Essentially correct if a scatterplot is given that includes axis labels and scales.

Partially correct if a scatterplot is given but only one of (i) the axis labels or (ii) the scales is included.

Incorrect if neither labels or scales are included.

Parts (b) and (c) should be read "collectively."

Part (b) is considered

Essentially correct if the interpretation of the correlation coefficient includes both strong and positive, and is interpreted in context.

Partially correct if the interpretation mentions two of the components: strong, positive, and context.

Parts (c) is considered

Essentially correct if comments are correct and are based on both the value of the correlation coefficient and the scatterplot.

Partially correct if it includes a correct comment based on only one of the components: correlation coefficient and the scatterplot.

Part (d) is considered

Essentially correct or **incorrect.**

NOTE: In order to recoup strength or direction for part (b) in part (c), the comments must be directly tied to the correlation coefficient. For example, saying the variables increase together does not count as saying the correlation tells us the association is positive. Interpretations in context in (c) can recoup missing context in (b).

NOTE: A construction such as "as x increases, y increases" counts as indicating a positive association, but does not by itself count as indicating linearity.

NOTE: A statement such as "judging by the scatterplot and correlation" without appealing to some characteristic of the plot or that the correlation is .85, is scored as partially correct.

Essentially correct responses count as 1 part and partially correct responses count as ½ part. If a paper is between two scores (for example 2 ½ parts), use a holistic approach to determine whether to score up or down depending on the strength of the response and communication.

Question 1 (cont'd.)

4 **Complete Response**
Four parts correct.

3 **Substantial Response**
Three parts correct.

2 **Developing Response**
Two parts correct.

1 **Minimal Response**
One part correct.

FR. Practice #2

Question 3

Solution

Part (a):

Yes, the linear model is appropriate for these data. The scatterplot shows a strong, positive, linear association between the number of railcars and fuel consumption, and the residual plot shows a reasonably random scatter of points above and below zero.

Part (b):

According to the regression output, fuel consumption will increase by 2.15 units for each additional railcar. Since the fuel consumption cost is \$25 per unit, the average cost of fuel per mile will increase by approximately $\$25 \times 2.15 = \$53.75$ for each railcar that is added to the train.

Part (c):

The regression output indicates that $r^2 = 96.7\%$ or 0.967. Thus, 96.7% of the variation in the fuel consumption values is explained by using the linear regression model with number of railcars as the explanatory variable.

Part (d):

No, the data set does not contain any information about fuel consumption for any trains with more than 50 cars. Using the regression model to predict the fuel consumption for a train with 65 railcars, known as extrapolation, is not reasonable.

Scoring

Each part is scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is essentially correct (E) if the model is deemed appropriate AND the explanation clearly indicates:

- There is a linear pattern in the scatterplot; OR
- There is no pattern in the residual plot.

Part (a) is partially correct (P) if the:

- Model is deemed appropriate AND the student refers to the scatterplot or residual plot but fails to state the relevant characteristic of the plot; OR
- Student refers to the relevant characteristic of the scatterplot or residual plot without deeming model appropriate.

Part (a) is incorrect (I) if the student:

- States that the model is appropriate without an explanation; OR
- States that the model is inappropriate; OR
- Makes a decision based only on numeric values from the computer output.

Question 3 (continued)

Part (b) is essentially correct (E) if the point estimate for the slope (2.15 or 2.1495) and the fuel consumption cost per unit ($25) are used to calculate the correct point estimate ($53.75 or $53.7375 ≈ $53.74).

Part (b) is partially correct (P) if only the point estimate for the slope (2.15 or 2.1495) is stated with a supporting calculation or interpretation.

Part (c) is essentially correct (E) if the student states:

- 96.7% of the variation in fuel consumption is explained by the linear regression model; OR
- 96.7% of the variation in fuel consumption is explained by the number of railcars.

Part (c) is partially correct (P) if the student makes one of the above statements using R-Sq(adj) = 96.3%.

Part (d) is essentially correct (E) if the student states that this is unreasonable due to extrapolation.

Part (d) is partially correct (P) if the student states this is:

- Unreasonable but provides a weak explanation; OR
- Reasonable even though it is considered a slight extrapolation.

Note: Any answer appearing without supporting work is scored as incorrect (I).

Each essentially correct (E) response counts as 1 point, each partially correct (P) response counts as ½ point.

4 Complete Response

3 Substantial Response

2 Developing Response

1 Minimal Response

Note: If a response is in between two scores (for example, 2 ½ points), use a holistic approach to determine whether to score up or down depending on the strength of the response and communication.

FR Practice 3

Question 6 (Investigative Task) Scoring Guidelines

4 Complete Response

I. Correctly estimates the asking price in dollars for at least two of the three models, including back transforming the predicted value for at least one of models b or c.

a. Price = -58.1 + 0.719(88) = 5.172
asking price is $5,172

b. Ln(price) = -14.9 + 0.185(88) = 1.380
exp(1.38) = 3.9749
asking price = $3,975

c. Sqrt(price) = -13.3 + 0.176(88) = 2.188
$(2.188)^2 = 4.7873$
asking price = $4,787

II. Describes the major shortcoming to be the non-linear pattern in the scatterplot or residual plot for all three models.

III. Suggests a new model that successfully deals with the non-linearity of the data. The prime contenders for this model are:

Fit a simple linear model after the first two or three years are dropped.
($R^2 = .978$ with little pattern in the residuals after dropping 79, 82 and 84.)

Fit separate linear models to the earlier and later years.
($R^2 = .985$ for the years 86 to 93, with little pattern in the residuals.)

Fit a model that attempts to model the curvature in the data. For example, fit a quadratic model to all of the data.
($R^2 = .974$ with little pattern in the residuals for a quadratic model.)

IV. Justifies why this model is better. For example, comments that there is no pattern in the residuals as seen from looking at the fitted model on the scatterplot or from looking at the residual plot.

3 Substantial Response

Fails to do one of the following satisfactorily:

- Part I (for example, by failing to back transform the prediction)
- describe the major shortcomings of the three models
- justify why the new model is better (for example, by failing to comment on the residual plot)

2 Developing Response

Fails to do one of the following satisfactorily:

- Part I and describe the major shortcomings of the three models
- Part I and justify why the new model is better
- suggest a new model and justify it
- describe the major shortcomings of the three models and justify why the new model is better.

1 Minimal Response

Gives correct responses to one of the items I, II, or III above. **Note:** Students cannot get IV correct without first specifying a new model.

Additional Free Response Questions (There will only be three on the test).

4.

The 1970 draft lottery involved matching birthdates with a number from 1 to 366. The lower the number, the more likely the individual with the matching birthday was to be drafted to fight in Vietnam. The average selection numbers by month are given in the following table.

MONTH	AVERAGE NUMBER
January	201.2
February	203.0
March	225.8
April	203.7
May	208.0
June	195.7
July	181.5
August	173.5
September	157.3
October	182.5
November	148.7
December	121.5

The following is part of the computer output for the least-squares regression line for predicting draft number from birth month (January = 1, February = 2, etc.).

Predictor	Coef	St Dev	*t* ratio	*P*
Constant	229.402	9.466	24.23	.000
Month	–7.057	1.286	–5.49	.000

a. What is the equation of the least-squares regression line for predicting average draft number from birth month?

$\widehat{\text{Number}} = 229.402 - 7.057$ (birth month)

b. Interpret the slope of the regression line in the context of the problem.

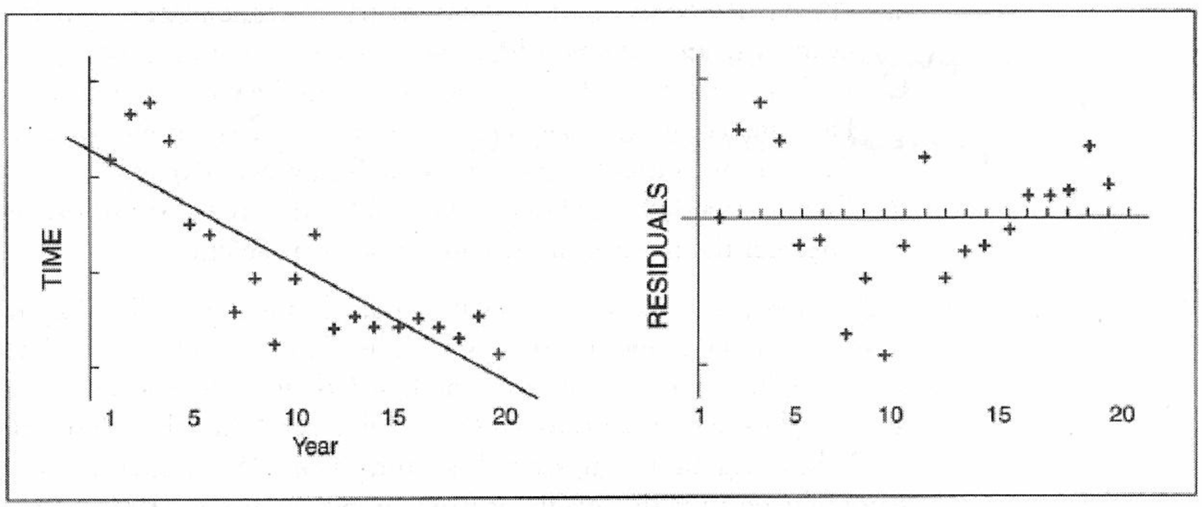
Each month from January to December the average draft number is predicted to decrease by 7.057

c. Does the computer analysis indicate that there is a useful (predictive) relationship between birth month and average draft number? Explain.

d. Construct a 95% confidence interval for the true slope of the regression line and interpret the interval in the context of the problem.

5.

The graphs below give the times of the winner of the women's race at the Boston Marathon from 1975 to 1994. The graph on the left is the scatterplot of the year (numbered from 1 so that 1975 = 1, 1976 = 2, etc.) versus the times (which range from 2 hours 48 minutes down to 2 hours, 22 minutes). The graph on the right is a plot of the residuals versus the year. The equation of the regression line is *Time* = 163.1 − 1.23(*Year*), where the year is the number of years after 1975.



a. What would be the predicted time for the winner in 1999 (that would be year 25)?

$\widehat{Time} = 163.1 - 1.23(25) = 132.32$

About 2 hours 12 minutes.

b. The winner in 1999 actually ran the race in 143 minutes. What is the residual for this prediction?

Residual = actual - predicted = 143 - 132 = 11 min

c. Does a line appear to be a good model for the data? Explain.

Not really: pattern in residual

d. If your goal was to predict the time for the winner in 1999, suggest an improvement to the situation described above but using the same data, or part of the same data.

? Maybe start the data at year 5 instead of year 1.

6.

Scientists have suspected that animals, when deficient in certain chemicals, tend to ingest natural resources that have high concentrations of those chemicals to offset the deficiency. In a study, scientists drained saliva from the parotid gland of sheep in order to make them sodium deficient. Then they offered these sheep a solution of sodium bicarbonate and measured the sheep's sodium intake. The sodium deficiency and the sodium intake, both measured in millimoles, are recorded as follows:

Sodium Deficit (in millimoles)	Sodium Intake (in milimoles)
100	110
200	180
570	610
850	790
700	750
425	390
375	420
325	380
450	300
850	790

The summary statistics and the regression output for this data are as follows:

Variable	N	Mean	Median	StDev	Q1	Q3
Deficit	10	484.5	437.5	256.4	293.7	737.5
Intake	10	472.0	405.0	250.0	270.0	760.0

Predictor	Coef	StDev	T	P
Constant	15.55	47.94	0.32	0.754
Deficit	0.94211	0.08843	10.65	0.000

(a) Does a line appear to be a reasonable model for this data? Explain your answer.

Scatter plot

residual plot

(b) State and interpret the slope in terms of the problem.

b = 0.94211

For every millimole of deficiency in sodium the sheep will take approximately .94211 millimoles of sodium

(c) Estimate the correlation between sodium deficit and sodium intake. Interpret your answer in the context of this problem.

r = .966

Strong, positive, linear.

(d) Estimate the amount of a sheep's sodium intake if the sheep is found to be deficient in sodium by 800 millimoles.

$\widehat{\text{Intake}} = 15.55 + 0.94211 (\text{deficit})$

$= 15.55 + 0.94211 (800)$

$=$ 769.238 millimoles

Practice 8

Question 5

Solution

Part (a):

Predicted Pulse = 63.457 + 16.2809 (Speed)

Part (b):

The intercept (63.457 bpm) provides an estimate for John's mean resting pulse (walking at a speed of zero mph).

The slope (16.2809 bpm/mph) provides an estimate for the mean increase in John's heart rate as his speed is increased by one mile per hour.

Part (c):

The margin of error for the confidence interval for the slope parameter is $t^*_{n-2} \times s_b$, where s_b is the standard error of the slope parameter. For a 98% confidence interval, the margin of error is $3.365 \times 0.8192 = 2.7566$ bpm.

Scoring

Part (a) is scored as essentially correct (E) or incorrect (I). Parts (b) and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Note: If the student uses *X* and *Y*, then both variables must be identified.

Part (b): There are four steps to constructing correct interpretations:

Step 1: A correct mathematical interpretation of the reported slope (16.2809) as a rate of increase in heart rate as walking speed increases.

Step 2: A correct mathematical interpretation of the reported intercept as a pulse rate when walking speed is zero.

Step 3: Correct use of units of measurement, e.g., John's heart rate increases 16.2809 bpm as his speed is increased by one mile per hour.

Step 4: Interpretation of the reported values as estimates of the corresponding mean quantities.

Question 5 (continued)

Part (b) is essentially correct (E) if all four steps are correct.

Part (b) is partially correct (P) if two or three steps are correctly addressed. Step 2 is scored as incorrect, for example, if the student suggests that the intercept does not have a meaningful interpretation.

Part (b) is incorrect (I) if at most one step is correct.

Note: The student is only penalized once for switching the variables.

Part (c) is essentially correct (E) if the standard error of the slope is identified and the correct critical value is used to calculate the margin of error.

Part (c) is partially correct (P) if the student:

- Computes the 98% confidence interval but does not identify the margin of error; OR
- Recognizes that the margin of error consists of the standard error of the coefficient and the critical value but uses an incorrect value for one of the two components or uses a t-value with 6 degrees of freedom and an incorrect standard error.

Part (c) is incorrect (I) if the student uses:

- The standard error of the coefficient as the margin of error; OR
- A critical value as the margin of error.

4 **Complete Response (3E)**

All three parts essentially correct

3 **Substantial Response (2E 1P)**

Two parts essentially correct and one part partially correct

2 **Developing Response (2E 0P or 1E 2P)**

Two parts essentially correct and zero parts partially correct
OR
One part essentially correct and two parts partially correct

1 **Minimal Response (1E 1P or 1E 0P or 0E 2P)**

One part essentially correct and either zero parts or one part partially correct
OR
Zero parts essentially correct and two parts partially correct

Practice 9

Question 2

Intent of Question

The primary goal of this question is to assess a student's ability to identify the estimated regression line and to identify and interpret important statistics from regression output provided by statistical software in the context of a practical problem.

Solution

Part (a):

The regression line is $\hat{y} = -2.679 + 9.5x$, where $\hat{y}$ represents the estimated (or predicted) mean height of the soapsuds and x represents the amount of detergent added to the pan.

Part (b):

The value $s = 1.99821$ mm is the standard deviation of the residuals. This statistic measures a typical amount of variability in the vertical distances from the observed height of the soapsuds to the regression line.
OR
The value $s = 1.99821$ mm is a measure of variation in the height of soapsuds for a given amount of detergent.

Part (c):

The standard error of the estimated slope parameter is 0.7553 mm per gram. Thus, the standard deviation of the estimated slope for predicting the height of soapsuds by using an amount of detergent is estimated to be 0.7553 mm per gram. This value estimates the variability in the sampling distribution of the estimated slope (i.e., how much we would expect sample slopes to vary from experiment to experiment).

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is essentially correct (E) if the least squares regression line is correctly identified and the variables are correctly defined.

Part (a) is partially correct (P) if:
the least squares regression line is correctly identified and either of the two variables are not correctly defined;
OR
the least squares regression line is not presented using estimated or predicted notation, or $\hat{y}$, AND both variables are correctly defined;
OR
only one of the two values is correctly identified from the table and both variables are correctly defined.

Part (a) is incorrect (I) if the least squares regression line is incorrectly identified or not identified, and the variables are not correctly defined.

Question 2 (continued)

Notes:

- If y is identified as the height of the soapsuds and x is identified as the amount of detergent, then the student should get credit for defining the variables. However, y must be identified as an estimated height somewhere in the student response in order to get this part essentially correct.
- If $\hat{y}$ (or *estimated height*) is used to specify the regression line and y is identified as the height, the response should be scored as essentially correct. For example, a response of $\text{estimated height} = -2.679 + 9.5\text{amount}$, where y represents the height of suds an x represents the amount of detergent should be scored essentially correct.
- If the regression line is specified using y (or height) instead of $\hat{y}$ (or estimated height), the response is scored as partially correct. For example, a response of $y = -2.679 + 9.5x$ where y = height of suds and x = amount of detergent should be scored as partially correct.
- If the estimates of the intercept and slope are reversed ($\hat{y} = 9.5 - 2.679x$), then the response should be scored as incorrect.

Part (b) is essentially correct (E) if the standard deviation is correctly interpreted in the context of this study.

Part (b) is partially correct (P) if the standard deviation is correctly interpreted in general terms without context.

Part (b) is incorrect (I) if the response indicates that s is any other standard deviation (e.g., univariate distribution of y), or slope.

Notes:

- If s is interpreted as the estimated standard deviation of the differences between the observed values for the height of soapsuds and the values predicted from the regression line, the response should be scored essentially correct.
- If s is interpreted as a "typical" prediction error for estimating height from the amount of detergent, then the response should be scored essentially correct.

Part (c) is essentially correct (E) if the standard error is identified and interpreted correctly.

Part (c) is partially correct (P) if standard error is identified but interpretation is weak (e.g., the standard error is a standard deviation of the slope). The major idea of sampling variability is not included.

Part (c) is incorrect (I) if the standard error is not correctly identified, identified with no interpretation, or an incorrect interpretation is provided.

Question 2 (continued)

4 **Complete Response**

All three parts essentially correct

3 **Substantial Response**

Two parts essentially correct and one part partially correct

2 **Developing Response**

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and two parts partially correct

OR

Three parts partially correct

1 **Minimal Response**

One part essentially correct and either zero or one part partially correct

OR

No parts essentially correct and two parts partially correct

Practice 10

Free-Response Scoring Guidelines: Question 4

4 Complete Response

(a) Correctly gives equation of the regression line as $\hat{y} = -20.5893 + 24.3929x$ (Could use y.) and defines both variables: x = # of teaspoons of weed killer; $\hat{y}$ = % killed
OR
percent killed $= -20.5893 + 24.3929$(# of teaspoons of weed killer)

(b) Substitutes $x = 2.6$ into the regression equation to get a predicted value of 42.83224, and notes that the residuals around the predicted value of 42.8 (or the middle of the predicted values) are negative. Concludes that since the residual for this prediction is negative, the prediction is expected to be too large.
OR
Notes that $x = 2.6$ is about in the middle of the explanatory values and, hence, the predicted percent killed will be close to the middle of the predicted values. Concludes that since the residual for this prediction is negative, the prediction is expected to be too large.
OR
Notes that $x = 2.6$ is in the middle of the explanatory values and that the residuals as a function of the explanatory values must exhibit the same pattern of positive and negative residuals. Since the residuals in the middle of the explanatory values are negative, the predicted value is expected to be too large.

- Arithmetic errors in (b) that give reasonable predictions (i.e. predictions between 20 and 60) should not be penalized.

3 Substantial Response

Gives a correct answer for either parts (a) or (b) and a partially correct answer to the other part.

Partially correct answers include but are not limited to:

- (a) Gives the correct equation but fails to define both variables.
- (a) Switches the values for slope and y-intercept in the equation but defines both variables.
- (a) Defines both variables but gives only one correct coefficient in the correct place of the linear equation.
- (b) Correctly explains why the residual at $x = 2.6$ is negative, but incorrectly interprets this negative residual to mean that the predicted value will be too small.
- (b) States that the residual at $x = 2.6$ is negative and thus the predicted value will be too large but fails to specify where the predicted residual for $x = 2.6$ is relative to the other residuals on the residual plot.
- (b) Uses the correct model and gets an incorrect prediction that is not reasonable, but reasons correctly using this prediction. (Unreasonable predictions are below 20 or greater than 60.)
- (b) Gives incorrect residual but interprets it correctly.

2 Developing Response

Gives a correct answer to one of (a) or (b) but not a partially correct answer to the other
OR
gives a partially correct response to both (a) and (b).

The following responses receive scores of zero for (b):

- Saying that the line cannot be used for prediction or saying that a prediction cannot be based on the computer output because the appropriate model should be quadratic or because the residuals have a non-random pattern. (An r^2 of 97.2% indicates that the fit is good; the residual plot merely reveals that the fit could be improved.)
- Computing a prediction (usually 42.8%) from the equation in (a) and saying that the prediction is "too large" without giving any reasoning from the prediction value to the conclusion of "too large."

1 Minimal Response

Gives a partially correct answer to either (a) or (b), but not both.
