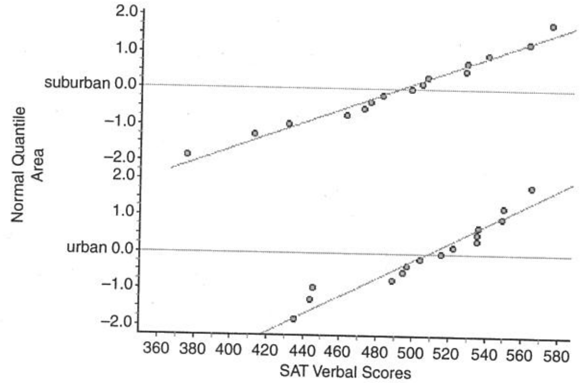

Since the conditions are reasonable, a two-sample t-test is appropriate.

Hypotheses:

$H_0: \mu_U = \mu_S$ or The mean SAT* Verbal scores for the two areas are the same.

$H_a: \mu_U \neq \mu_S$ or The mean SAT* Verbal scores for the two areas are different.

Mechanics:

$n_U = 15$ $n_S = 15$

$\bar{x}_U \approx 507.8$ $\bar{x}_S \approx 490.3$

$s_U \approx 40.76$ $s_S \approx 55.21$

$$t_{df=25.8} \approx \frac{(507.8 - 490.3) - 0}{\sqrt{\frac{(40.76)^2}{15} + \frac{(55.21)^2}{15}}} \approx 0.99$$

p-value ≈ 0.33

Conclusion:

At any reasonable α-level with such a high p-value (0.33), we fail to reject the null hypothesis. There is insufficient evidence to show a statistically significant difference in the average SAT* Verbal scores for urban and suburban areas.

Answers to Review Questions for Topic 7

Multiple Choice

1. B

   χ^2 tests are for categorical data. A χ^2 test of independence examines the distribution of counts for one group of individuals classified according to two categorical variables.

2. C

   $$\text{Expected Value} = \frac{(\text{row total})(\text{column total})}{\text{table total}}$$

   $$\text{Expected Value (First-Class and Dead)} = \frac{(2201 - 710)(325)}{2201} \approx 220.2$$

3. E

χ^2 Goodness-of-fit test

	Monday	Tuesday	Wednesday	Thursday	Friday
Observed	98	84	92	86	76
Expected	(87.2)	(87.2)	(87.2)	(87.2)	(87.2)
$\frac{(O-E)^2}{E}$	$\frac{(98-87.2)^2}{87.2}$	$\frac{(84-87.2)^2}{87.2}$	$\frac{(92-87.2)^2}{87.2}$	$\frac{(86-87.2)^2}{87.2}$	$\frac{(76-87.2)^2}{87.2}$

$$\chi^2_{df=4} = \sum_{all} \frac{(\text{observed} - \text{expected})^2}{\text{expected}} \approx 3.17$$

p-value ≈ 0.529

4. C

The computer output gives the p-value for a *two*-tailed test as 0.0279. Since we are doing a *one*-tailed test, we need to divide this value in half.

5. B

95% confidence interval for slope:

$b \pm t^*_{df=n-2}\,\mathrm{SE}_{(b)}$

$0.00733 \pm 2.086(0.0031)$

```
Dependent variable is     SAT-Math
No Selector
R squared = 21.9%    R squared (adjusted) = 18.0%
s = 27.99 with 22 - 2 = 20 degrees of freedom

Source        Sum of Squares    df    Mean Square    F-ratio
Regression       4401.65         1       4401.65       5.62
Residual        15666.9         20        783.347

Variable       Coefficient      s.e. of Coeff    t-ratio      prob
Constant        446.990          37.66            11.9       ≤ 0.0001
Per Student     7.33145e-3       0.0031            2.37        0.0279
```

slope   SE(slope)   df (slope)

6. D

Hypotheses are always stated in terms of the parameter of interest. The parameter of interest is the population slope β (*not* the statistic b).

7. D

The fan-shaped pattern in the residual plot indicates that as the value of x increases, the model is less accurate in predicting the corresponding y-value, so choice A is correct. The residuals are close to the line $y = 0$ for small values of x, but the residuals are larger as x increases, so choice B is also correct.

Either a curved pattern or unequal variance in the residuals suggests that a nonlinear model may fit the data better than a linear model. In this case we see unequal variance (a fan-shaped pattern), so C is incorrect.

8. E

   The appropriate test is a χ^2 test of homogeneity (equal proportions). The data are given as counts, and there are two separate groups (faculty and administrators) with the same variable (response to the question asked) measured on each group. Although the surveyed individuals may not constitute random samples, this is not a problem because we are not trying to generalize to a larger group.

9. A

   For a χ^2 test of homogeneity (equal proportions), the degrees of freedom are $(rows - 1)(columns - 1) = (1)(2) = 2$. (Be careful not to include the totals when you count the number of rows and columns.)

10. E

    The p-value is approximately 0, lower than any acceptable α-level, so there is ample evidence that at least one of the proportions, and therefore one response group, is significantly different. (It is not enough for table values to be different; they must be *significantly* different. You MUST perform a statistical test.)

Free Response

Hypotheses:

H_0: The booking methods used by customers of this hotel in 1995, 2000, and 2005 have the same distribution (are homogeneous).

H_a: The booking methods used by customers of this hotel in 1995, 2000, and 2005 do not have the same distribution.

Assumptions/Conditions for χ^2 Test of Homogeneity

Counted data condition: I have counts of the number of customers in categories.

Randomization condition: I don't want to draw inferences to other hotels, so there is no need to check for a random sample.

Expected cell-frequency condition: The expected values (shown in the table in parentheses) are all at least 5.

	1995	2000	2005
Travel Agent	112 (90.7)	103 (90.7)	57 (90.7)
Book Online	42 (80.7)	77 (80.7)	123 (80.7)
Other	56 (38.7)	30 (38.7)	30 (38.7)

Since the assumptions and conditions are reasonable, we will proceed with a χ^2 test of homogeneity. (We choose a test of homogeneity because there are three separate samples—one for each year.)

Mechanics:

$$\chi^2_{df=4} = \sum_{all} \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

$\chi^2_{df=4} \approx 71.7711$
$p\text{-value} \approx 0$

Conclusion:

Since the p-value is essentially zero, I reject the null hypothesis at any reasonable α-level and conclude that the booking methods chosen by customers of this hotel have changed over the years examined.

ANSWERS TO PRACTICE EXAMINATION 1

Section I: Multiple Choice

1. B

   IQR = Q3 − Q1. Using the 1.5 IQR Rule:

   Q1 − 1.5 IQR = 119.5 − 1.5(16) = 95.5, which is less than the minimum value *and* Q3 + 1.5 IQR = 135.5 + 1.5(16) = 159.5, which is less than the maximum value. Therefore, there is at least one outlier, the maximum value.

2. D

   $P(A) + P(B) = 0.69$; $P(A \cup B) = 0.68$; $0.69 \neq 0.68$ and thus the two events are not mutually exclusive.

   $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.69 - P(A \cap B) = 0.68$; therefore, $P(A \cap B) = 0.01$.

   The two events are not independent. If they were, then either $P(B \mid A) = P(B)$ *or* $P(A)P(B) = P(A \cap B)$ would have to be true. But,
   $$P(B \mid A) = \frac{P(A \cap B)}{(A)} = \frac{0.01}{0.43} = 0.023 \text{ which} \neq P(B), 0.26. \text{ And}$$
   $P(A)P(B) \neq P(A \cap B)$ since $0.11118 \neq 0.01$.

3. C

   The current belief is that the average family income is \$45,000. Thus the null hypothesis should set μ equal to 45,000. The first-time home buyers are trying to show that the average family income is less than \$45,000; therefore, the alternative hypothesis should set μ less than 45,000.

4. A

   With a mean of 14.1 and a standard deviation of 0.04, the curve should have a peak at 14.1, and the change in curvature should occur 0.04 unit on either side of 14.1. *At least 14 oz* means 14 oz or more. Therefore, the shading should be to the right of 14.

5. D

   The upper quartile of set Y is equivalent to the median of set X. Therefore, approximately 50% of the data values in set X are greater than approximately 75% of the data values in set Y.

6. C

   The data are blocked by grocery item; therefore, the matched-pairs design is appropriate for this test.