Answer Key

1. **E**	11. **D**	21. **E**	31. **A**	41. **D**
2. **E**	12. **B**	22. **D**	32. **E**	42. **E**
3. **B**	13. **B**	23. **D**	33. **E**	43. **B**
4. **A**	14. **D**	24. **A**	34. **B**	44. **B**
5. **B**	15. **B**	25. **C**	35. **D**	45. **B**
6. **B**	16. **D**	26. **E**	36. **B**	46. **B**
7. **D**	17. **E**	27. **A**	37. **C**	47. **A**
8. **D**	18. **B**	28. **E**	38. **A**	
9. **C**	19. **A**	29. **D**	39. **D**	
10. **E**	20. **D**	30. **A**	40. **C**	

Answers Explained

Multiple-Choice

1. **(E)** There is no reason to assume that the probability of getting a 5 is the same as that of not getting a 5.

2. **(E)** The probability that *both* events will occur is the product of their separate probabilities only if the events are independent, that is, only if the chance that one event will happen is not influenced by whether or not the second event happens. In this case the probability of different surgeries failing are probably closely related.

3. **(B)** $P(\text{1st die is 4})P(\text{2nd die is 6}) + P(\text{1st die is 6})P(\text{2nd die is 4}) + P(\text{1st die is 5})P(\text{2nd die is 5})$

4. **(A)** $10(.15)^2(.85)^3 = .138$ or binompdf(5, .15, 2) = .138

5. **(B)** The probability of the next child being a girl is independent of the gender of the previous children.

6. **(B)** $1 - (.99975)^{100} = .0247$

7. **(D)** $.001(1{,}500{,}000) = 1500$

8. **(D)** $\bar{x} = E(X) = \sum xP(x) = 0(.999) + 25{,}000(.001) = 25$

$$\sigma = \sqrt{\sum (x - \bar{x})^2 P(x)} = \sqrt{(0-25)^2(.999) + (25{,}000 - 25)^2(.001)} = 790.17$$

[Or on the TI-84, put returns and probabilities into two lists and run `1-Var Stats L1,L2`]

9. **(C)** $\frac{1500}{6000} = .25$ are honors students, and $\frac{1800}{6000} = .3$ prefer basketball. Because of independence, their intersection is $(.25)(.3) = .075$ of the students, and $.075(6000) = 450$.

10. **(E)** $(.9)^3 + 3(.9)^2(.1) = .972$

11. **(D)** $P(\text{famine} \mid \text{plague}) = \dfrac{P(\text{famine} \cap \text{plague})}{P(\text{plague})} = \dfrac{.15}{.39} = .385$

12. **(B)** If E and F are independent, then $P(E \cap F) = P(E)P(F)$; however, in this problem, $(.4)(.35) \neq .3$.

13. **(B)** Because A and B are independent, we have $P(A \cap B) = P(A)P(B)$, and thus $P(A \cup B) = .2 + .1 - (.2)(.1) = .28$.

14. **(D)** $\dfrac{5{,}592{,}012}{9{,}664{,}994} = .579$

15. **(B)** $E(X) = \mu_x = \Sigma x_i p_i = 700(.05) + 540(.25) + 260(.7) = 352$

16. **(D)** The probability of throwing heads is .5. By the law of large numbers, the more times you flip the coin, the more the relative frequency tends to become closer to this probability. With fewer tosses there is a greater chance of wide swings in the relative frequency.

17. **(E)** P(diamond on 2nd pick) = 1/4; however,
P(diamond on 2nd pick | heart on first pick) = 13/51
P(black on 2nd pick) = 1/2; however, P(black on 2nd pick | red on 1st pick) = 26/51
P(ace on 2nd pick) = 1/13; however, P(ace on 2nd pick | ace on 1st pick) = 3/51
P(two kings) = (1/13)(3/51); however, P(two kings | two aces) = 0
P(black on 2nd pick) = 1/2 and P(black on 2nd pick | two aces) = 6/12

18. **(B)** P(two kings) = (1/13)(1/13); however, P(two kings | two aces) = 0
P(king on 2nd pick) = 1/13, and P(king on 2nd pick | ace on 1st pick) = 1/13
P(at least one king) = $1 - (12/13)^2 = 25/169$; however,
P(at least one king | at least one ace) =
$[(1/13)(1/13) + (1/13)(1/13)]/[25/169] = 2/25$
P(exactly one king) = $(1/13)(12/13) + (12/13)(1/13) = 24/169$; however,
P(exactly one king | exactly one ace) = $(2/169)/(24/169) = 1/12$
P(no kings) = $(12/13)^2$; however, P(no kings | no aces) = $(11/12)^2$

19. **(A)** $E(X) = \mu_x = \Sigma x_i p_i = 21{,}000(.8) + 35{,}000(.2) = 23{,}800$

20. **(D)** $1000(.2) + 5000(.05) = 450$, and $800 - 450 = 350$.

21. **(E)** $5\left(\frac{4}{5}\right) + 100\left(\frac{1}{5}\right) = 24$, and $5\left(\frac{400}{500}\right) + 100\left(\frac{100}{500}\right) = 24$

22. **(D)** $P(E \cap F) = P(E)P(F)$ only if the events are independent. In this case, women live longer than men and so the events are not independent.

For questions 23–27, we first sum the rows and columns:

	GPA <2.0	2.0–3.0	>3.0	
Many skipped classes	80	25	5	110
Few skipped classes	175	450	265	890
	255	475	270	1000

23. **(D)** $\frac{25+450}{1000} = .475$

24. **(A)** $\frac{80}{1000} = .080$ (probability of an intersection)

25. **(C)** $\frac{80+25+5+175}{1000} = .285$ (probability of a union)

26. **(E)** $\frac{80}{80+25+5} = .727$ (conditional probability)

27. **(A)** $P(2.0 - 3.0\ \text{GPA}) = \frac{475}{1000} = .475$; however, $P(2.0 - 3.0\ \text{GPA} \mid \text{few skips}) = \frac{450}{890} = .506$. If independent, these would have been equal.

28. **(E)** While the outcome of any single play on a roulette wheel or the age at death of any particular person is uncertain, the law of large numbers gives that the relative frequencies of specific outcomes in the long run tend to become closer to numbers called probabilities.

29. **(D)** $\frac{36+22}{36+22+5+2} = .892$

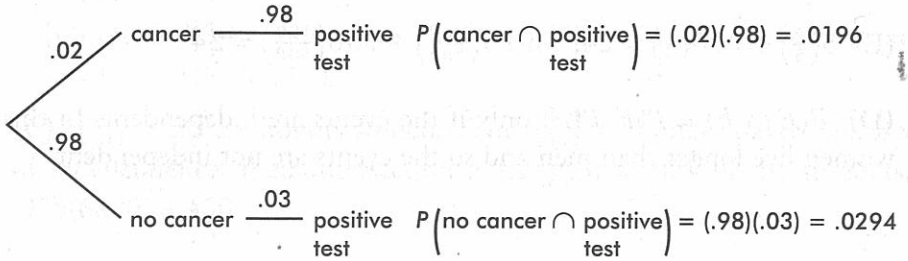
30. **(A)** $\frac{35}{500} = .070$

31. **(A)** $\frac{25}{460} = .054$

32. **(E)** $\frac{35}{40} = .875$

33. **(E)** $\frac{435}{460} = .946$

34. **(B)**



$$\begin{aligned} P(\text{positive test}) &= P(\text{cancer} \cap \text{positive test}) \\ &\quad + P(\text{no cancer} \cap \text{positive test}) \\ &= .0196 + .0294 \\ &= .0490 \end{aligned}$$

$$P(\text{cancer} \mid \text{positive test}) = \frac{P(\text{cancer} \cap \text{positive test})}{P(\text{positive test})}$$

$$= \frac{.0196}{.0490}$$

$$= .4$$

35. **(D)** The probability that you will receive an A in AP Statistics but not in AP Biology must be .35 – .19 = .16, not .17.

36. **(B)** Your expected winnings are only

$$\frac{1}{7{,}500{,}000}(1{,}000{,}000) + \frac{5}{7{,}500{,}000}(10{,}000) + \frac{20}{7{,}500{,}000}(100) = 0.14$$

37. **(C)** Even though the first two choices have a higher expected value than the third choice, the third choice gives a 100% chance of avoiding bankruptcy.

38. **(A)** The probabilities $\frac{7}{24}$, $\frac{8}{24}$, and $\frac{9}{24}$ are all nonnegative, and they sum to 1.

39. **(D)** $6(.65)^2(.35)^2 = .311$ or binompdf(4, .65, 2) = .311

40. **(C)** This is a binomial with $n = 10$ and $p = .37$, and so the mean is $np = 10(.37) = 3.7$.

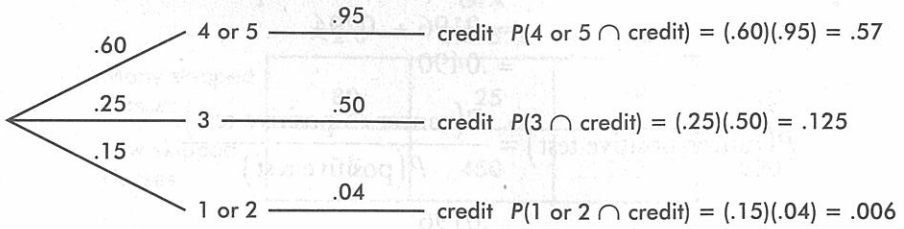
41. **(D)** $\frac{.40}{.40+.20+.15} = .533$

42. **(E)** Coins have no memory, and so the probability that the next toss will be heads is .5 and the probability that it will be tails is .5. The law of large numbers says that as the number of tosses becomes larger, the percentage of heads tends to become closer to .5.

43. **(B)** The probabilites of each pump not failing are 1 – .025 = .975, 1 – .034 = .966, and 1 – .02 = .98, respectively. The probability of none failing is (.975)(.966)(.98) = .923, and so the probability of at least one failing is 1 – .923 = .077.

44. **(B)** If $p > .5$, the more likely numbers of successes are to the right and the lower numbers of successes have small probabilities, and so the histogram is skewed to the left. No matter what p is, if n is sufficiently large, the histogram will look almost symmetric.

45. **(B)**



$$P(\text{credit}) = .57 + .125 + .006 = .701$$

$$P(3 \mid \text{credit}) = \frac{P(3 \cap \text{credit})}{P(\text{credit})} = \frac{.125}{.701} = .178$$

46. **(B)** If A and B are mutually exclusive, $P(A \cap B) = 0$. Thus $.6 = .4 + P(B) - 0$, and so $P(B) = .2$. If A and B are independent, then $P(A \cap B) = P(A)P(B)$. Thus $.6 = .4 + P(B) - .4P(B)$, and so $P(B) = \frac{1}{3}$.

47. **(A)** For each of the 20 students, we must generate an answer to each of the 10 questions, and record if any student has at least 6 out of 10 correct answers.

Free-Response

1. With Option A, if in reality only 75% of the articles meet all specifications, the probability of rejecting the day's production is:

$$P(\text{rejection}) = 1 - P(\text{acceptance}) = 1 - \left[\binom{6}{5}(.75)^5(.25) + (.75)^6\right] = .466$$

[On the TI-84, `binomcdf(6,.75,4) = .466`]

With Option B, if in reality only 75% of the articles meet all specifications, the probability of rejecting the day's production is:

$$P(\text{rejection}) = 1 - P(\text{acceptance}) =$$

$$1 - \left[\binom{12}{10}(.75)^{10}(.25)^2 + \binom{12}{11}(.75)^{11}(.25) + (.75)^{12}\right] = .609$$

[On the TI-84, `binomcdf(12,.75,9) = .609`]

For the greatest probability of rejecting the day's production if only 75% of the articles meet all specifications, the buyer should request the manufacturer to use Option B with a probability of rejection of .609 as opposed to Option A with a probability of rejection of only .466.